Aus der Abschätzung der Koeffizienten r_ν von $\mathcal{R}$ in Thm. 19 folgt ...

$\Rightarrow$ Error bound of $\langle \mathcal{R}_\Lambda(w_N), (1-J)v \rangle$ by $\left(\sum_{\nu\in\Lambda} \eta_\nu^2(w_N)\right)^{1/2}$

Next, for the <u>truncation</u> of the residual $\mathcal{R}_{\partial\Lambda}(w_N)$, by Parseval,

$$\| \mathcal{R}_{\partial\Lambda}(w_N)\|^2_{L^2_\pi(\Gamma;V^*)} = \sum_{\nu\in\partial\Lambda} \| r_\nu(w_N)\|^2_{V^*} .$$

Cauchy-Schwarz and triangle inequalities lead to

$$\| r_\nu(w_N)\|_{V^*} = \sup_{v\in V} \frac{1}{\|v\|_V} \left| \int_D \sigma_\nu(w_N)\cdot \nabla v \, dx \right| \le \zeta_\nu(w_N)$$

with $\zeta_\nu(w_N) := \sum_{m\ge 1} \left\| \frac{a_m}{a_0} \right\|_{L^\infty(D)} \left(\beta_{\nu_m+1} \|w_{N,\nu+\varepsilon_m}\|_V + \beta_{\nu_m} \|w_{N,\nu-\varepsilon_m}\|_V \right)$

$\Rightarrow$ <u>Lem. 11:</u> If $0\in\Lambda$, then for $w_N \in V_N$,

$$\| \mathcal{R}_{\partial\Lambda}(w_N)\|_{L^2_\pi(\Gamma;V^*)} \le \zeta(w_N, \partial\Lambda) := \left(\sum_{\nu\in\partial\Lambda} \zeta_\nu^2(w_N)\right)^{1/2}.$$

Combining the results leads to an <u>a posteriori error estimate</u> of the form

$$\|w_N - u\|_{\mathcal{A}} \lesssim \left(\sum_{\nu\in\Lambda} \eta_\nu^2(w_N)\right)^{1/2} + \left(\sum_{\nu\in\partial\Lambda} \zeta_\nu^2(w_N)\right)^{1/2} + \|w_N - u_N\|_{\mathcal{A}}$$

(+ neglected dimensions)

can be estimated by ζ_ν, see [Eos 2E] or ignore :)