19th Berlin-Oxford Young Researcher's Meeting on Applied Stochastic Analysis

24^{th} **June** – 26^{th} **June** 2024

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1. Welcome

It is our great pleasure to welcome you to the 19th Berlin-Oxford Young Researchers Meeting on Applied Stochastic Analysis. We hope you enjoy a productive meeting!

Conference organisers
Christian Bayer (WIAS Berlin)
Jana Bielagk (HU Berlin)
Peter Friz (TU Berlin)
Terry Lyons (University of Oxford)
Luca Pelizzari (WIAS Berlin)
Thomas Wagenhofer (TU Berlin)
Benjamin Walker (University of Oxford)

Presentations
All talks will be held in person. They will be 25 minutes and we will have 5 minutes for questions after each talk. On Tuesday will be a conference dinner at 18:30.

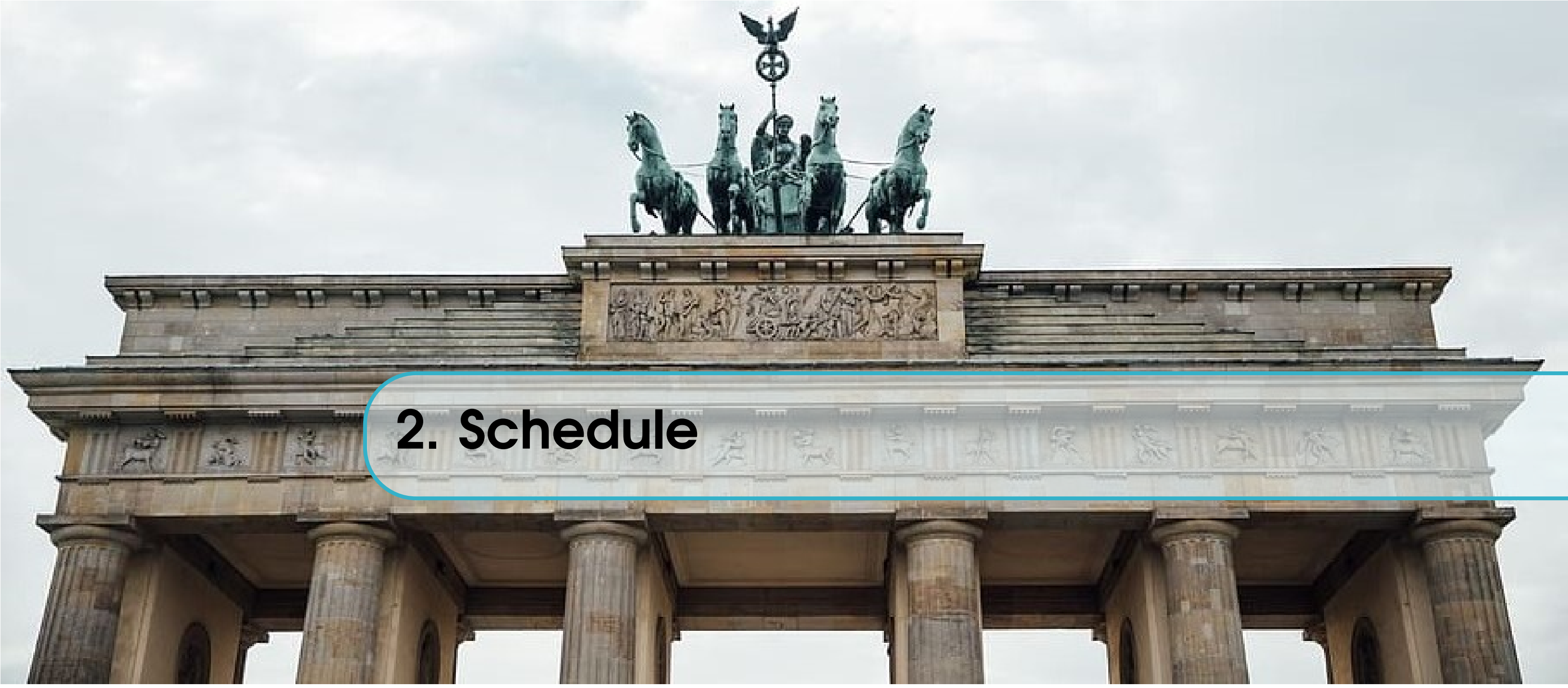
Supporting Institutions

DataSig

WIAS

Stochastic Analysis in Interaction

Gefördert durch DFG Deutsche Forschungsgemeinschaft



2. Schedule

Monday, 24th June
Location: WIAS Berlin

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12:00–13:30	**Lunch Break**		
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16:00–16:30	Marco Rehmeier (Bielefeld University)	*Probabilistic representation of nonlinear Fokker-Planck equations and nonlinear PDEs by nonlinear Markov processes*	12

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Location: WIAS Berlin

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10:00–10:30	Florian Bechtold (Bielefeld University)	*On Young regimes for locally monotone SPDEs*	8
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11:30–12:00	Shyam Popat (University of Oxford)	*Well-posedness of the Dean-Kawasaki equation with correlated noise on bounded domains*	10
12:00–13:30	**Lunch Break**		
13:30–14:00	Oleg Butkovsky (WIAS Berlin)	*Optimal weak uniqueness for SDEs driven by fractional Brownian motion and for stochastic heat equation with distributional drift.*	9
14:00–14:30	Gideon Chiusole (TU Munich)	*Towards Abstract Wiener Model Spaces*	9
14:30–15:00	Leander Schnee (FU Berlin)	*Equilibrium Fluctuations for Weakly Asymmetric Interacting Particle Systems with Boundaries*	12
15:00–15:30	**Coffee Break**		
15:30–16:00	Matthew Buckland (University of Oxford)	*Branching Interval Partition Diffusions*	8
16:00–16:30	Florin Suciu (Université Paris Dauphine-PSL)	*A gradient flow on control space with rough initial condition*	12
18:30	**Conference dinner**		

Wednesday, 26th June
Location: WIAS Berlin

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11:00–11:30	Leonard Schmitz (MPI MIS Leipzig)	*Free generators and Hoffman's isomorphism for the two-parameter shuffle algebra*	12
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3. Talks and Abstracts

3.1 On Young regimes for locally monotone SPDEs

Florian Bechtold, Bielefeld University

Tuesday 10:00-10:30

We consider the following SPDE on a Gelfand-triple (V,H,V^*):

$$\begin{aligned} du(t) &= A(t,u(t))dt + dI_t(u), \\ u(0) &= u_0 \in H. \end{aligned}$$

Given certain local monotonicity, continuity, coercivity and growth conditions of the operator $A : [0,T] \times V \to V^*$ and a sufficiently regular operator I we establish global existence of weak solutions.

In analogy to the Young regime for SDEs, no probabilistic structure is required in our analysis, which is based on a careful combination of monotone operator theory and the recently developed Besov rough analysis due to Friz and Seeger. Due to the abstract nature of our approach, it applies to various examples of monotone and locally monotone operators A, such as the p-Laplace operator, the porous medium operator, and an operator that arises in the context of shear-thickening fluids; and operators I, including additive Young drivers $I_t(u) = Z_t - Z_0$, abstract Young integrals $I_t(u) = \int_0^t \sigma(u_s)dX_s$, and translated integrals $I_t(u) = \int_0^t b(u_s - w_s)ds$ that arise in the context of regularization by noise. In each of the latter cases, we identify corresponding noise regimes (i.e. Young regimes) that assure our abstract result to be applicable. In the case of additive drivers, we identify the Brownian setting as borderline, i.e. noises which enjoy slightly more temporal regularity are amenable to our completely pathwise analysis.

3.2 Conditional Stochastic Optimal Control

Franziska Bielert, TU Berlin

Monday 14:30-15:00

We consider a stochastic optimal control problem with anticipative controls. This leads to a random path-dependent value function. We provide an HJB equation using the functional Itô formula. It includes a novel condition on the path-dependent derivatives of the value function.

3.3 Branching Interval Partition Diffusions

Matthew Buckland, University of Oxford

Tuesday 15:30-16:00

We construct an interval-partition-valued diffusion from a collection of excursions sampled from the excursion measure of a real-valued diffusion, and we use a spectrally positive Lévy process to order both these excursions and their start times. At any point in time, the interval partition generated is the concatenation of intervals where each excursion alive at that point contributes an interval of size given by its value. Previous work by Forman, Pal, Rizzolo and Winkel considers self-similar interval partition diffusions – and the key aim of this work is to generalise these results by dropping the self-similarity condition. The interval partition can be interpreted as an ordered collection of individuals (intervals) alive that have varying characteristics and generate new intervals during their finite lifetimes, and hence can be viewed as a class of Crump-Mode-Jagers-type processes.

3.4 Continuity and differentiability of rough stochastic differential equations with respect to a parameter

Fabio Bugini, TU Berlin

Monday
14:00-14:30

We study rough stochastic differential equations (SDEs) of the form

$$dX_t^\zeta \ = \ b_t(\zeta, X_t^\zeta)\, dt + \sigma_t(\zeta, X_t^\zeta)\, dB_t + \beta_t(\zeta, X_t^\zeta)\, d\mathbf{W}_t$$

where ζ is a parameter, B is a Brownian motion and $\mathbf{W}$ is a deterministic α-rough path with $\alpha \in (\frac{1}{3}, \frac{1}{2}]$. Our focus is to understand the dependence of the solution X on the parameter ζ. Specifically, we investigate under which assumptions on the coefficients we have continuity and/or differentiability in mean with respect to ζ. As an application, we show how our theory can be useful in providing an existence-and-uniqueness result for a class of linear backward rough partial differential equations (PDEs). Using a rough Feynman-Kac formula, we prove that solutions of such rough PDEs admit a functional integral representation in term of solutions of appropriate rough SDEs, where the initial time and the initial state play the role of parameters..

3.5 Optimal weak uniqueness for SDEs driven by fractional Brownian motion and for stochastic heat equation with distributional drift.

Oleg Butkovsky, WIAS Berlin

Tuesday
13:30-14:00

Joint work with Leonid Mytnik (Technion, Israel). We consider the stochastic differential equation

$$dX_t = b(X_t)dt + dB_t^H,$$

where the drift b is a Schwartz distribution lying in the space $\mathscr{C}^\alpha$, $\alpha < 0$, and B^H is a fractional Brownian motion of Hurst index $H \in (0, 1/2]$. If $H = 1/2$, then both weak and strong uniqueness theories for this SDE have been developed. However, the situation is much more complicated if $H \neq 1/2$, as the main tool, Zvonkin transformation, becomes unavailable in this setting. The breakthroughs of Catellier and Gubinelli, and later by Le, allowed to establish strong well-posedness of this SDE via sewing/stochastic sewing arguments. However, weak uniqueness of this SDE remained a challenge for quite a while, since a direct application of stochastic sewing alone does not seem to be very fruitful. I will explain how a combination of stochastic sewing with certain arguments from ergodic theory allows to show weak uniqueness in the whole regime where weak existence is known, that is $\alpha > 1/2 - 1/(2H)$. If time permits, we will discuss how one can get a weak rate of convergence of the corresponding Euler scheme and see how a similar argument yields weak uniqueness of stochastic heat equation with distributional drift also in the whole regime where weak existence is known.

3.6 Towards Abstract Wiener Model Spaces

Gideon Chiusole, TU Munich

Tuesday
14:00-14:30

Abstract Wiener spaces are in many ways the decisive setting for fundamental results on Gaussian measures: large deviations (Schilder), quasi-invariance (Cameron-Martin), differential calculus (Malliavin), support description (Stroock-Varadhan), concentration of measure (Fernique), ... Analogues of these classical results have been derived in the "enhanced" context of Gaussian rough paths and, more recently, regularity structures equipped with Gaussian models. The aim of this article is to propose a notion of "abstract Wiener model space" that encompasses the aforementioned. More specifically, we focus here on enhanced Schilder type results, Cameron-Martin shifts and Fernique estimates, offering a somewhat unified view on results in Friz-Victoir 2007 and Hairer-Weber 2015.

3.7 Novel Random Vortex Methods for approximating incompressible Navier-Stokes equations

Sebastian Ertel, TU Berlin

Wednesday
09:30-10:00

Random Vortex Methods are a class of Monte Carlo algorithms that aim to approximate the solution to the incompressible Navier-Stokes equation (NSE) using a system of interacting Stochastic Differential Equations.

Such methods have originally been introduced and mathematically analyzed for homogenous 2D NSE on the full plane, however extensions to more pracically interesting regimes such as the 3D case and for bounded domains have been developed in recent years. In this talk we focus on an extension to the inhomogenous setting, i.e. when the modelled fluid flow is influenced by an external force. We first derive a novel representation of the solution of the NSE by a field valued McKean–Vlasov equation, which allows for computationally efficient approximations. Next we show well posedness of this class of McKean–Vlasov equations. Finally we discuss quantitative convergence results for a class of numerical schemes using a combination of grid discretization and interacting particle systems. If time permits we also briefly discuss how Random Vortex Methods can be used to derive efficient Physics-informed Neural Networks for Machine Learning approaches in Computational Fluid Dynamics.

3.8 Well-posedness of the Dean-Kawasaki equation with correlated noise on bounded domains

Shyam Popat, University of Oxford

Tuesday
11:30-12:00

We extend the notion of stochastic kinetic solutions introduced in [Fehrman, Gess, '24] to establish the well-posedness of stochastic kinetic solutions of generalised Dean-Kawasaki equations with correlated noise on bounded, C^2 domains with Dirichlet boundary conditions. In this talk we will be brief with the setup and instead focus on explaining the main difficulty to extend the result of [Fehrman, Gess, 24'] on the torus to the bounded domain, both on the existence and the uniqueness sides.

3.9 Uniform Attraction and Metastability for Stochastic Damped Wave Equations

Ioannis Gasteratos, Imperial College London

Tuesday
09:30-10:00

We study a class of damped wave equations with non-Lipschitz nonlinearities on a bounded interval. The system is perturbed by multiplicative space-time white noise. Using tools from large deviations, we aim to establish logarithmic, small-noise asymptotics for exit times and exit places of solutions from neighborhoods of metastable states. First, we rigorously prove the existence of domains of uniform attraction for the noiseless system with continuous initial position and distributional initial velocity. Then, following Freidlin's classical work on the exit problem for finite-dimensional diffusions, we characterize the divergence rates of exit times in terms of the large deviations action functional. The proof highlights significant differences from both the finite-dimensional and parabolic analogues of the exit problem. This is joint work with Michael Salins and Konstantinos Spiliopoulos.

3.10 Advancing Optimal Control with Signatures

Paul Hager, TU Berlin

Monday
09:30-10:00

The role of signatures in solving non-Markovian control problems has been increasingly recognized, particularly in areas of mathematical finance, such as optimal execution, portfolio optimization, and the valuation of American options. In this work, we study a general class of differential equations driven by stochastic rough paths, where the control impacts the system's drift. In the theoretical aspect, we demonstrate that optimal controls can be approximated using linear and deep signature functionals. This includes a refined lifting result for progressively measurable processes into continuous path-functionals, in addition to implementing a robust stability result for rough differential equations from Diehl et al., 2017. Building on these theoretical insights, we have developed a practical numerical methodology based on Monte-Carlo sampling and deep learning techniques. We demonstrate the efficiency of this methodology through numerical examples, including the optimal tracking of fractional Brownian motion, for which we provide exact theoretical benchmarks.

3.11 Rough additive functions and application to Yang Mills theory

Abdulwahab Mohamed, University of Edinburgh

Tuesday
11:00-11:30

We introduce the concept of rough additive functions which is an extension of rough paths theory to line integrals of distributional 1-forms. In the context of gauge theory, we use this notion to define controlled gauge transformations and holonomies via RDEs. We briefly discuss a rough extension of Uhlenbeck compactness theorem on the unit square. The main ingredient is a singular elliptic SPDE to obtain a Coulomb gauge which we solve using regularity structures. Surprisingly, we manage to bound all singular terms occurring in the regularity structures by the “norm” of rough additive functions. As a consequence our result can be seen as the first gauge fixed representation of the Yang-Mills measure on the unit square using PDE techniques. This is a joint work with Ilya Chevyrev and Tom Klose.

3.12 Lookdown construction for a Moran seed-bank model

Julio Ernesto Nava Trejo, HU Berlin

Monday 15:30-16:00

In this talk, we present a lookdown construction for a Moran seed-bank model with variable active and inactive population sizes, and we show that the empirical measure of our model coincides with that of the Seed-Bank-Moran Model with latency. Furthermore, we prove that the time to the most recent common ancestor, starting from N individuals with stationary distribution over its state (active or inactive), has the same asymptotic order as the largest inactivity period. Additionally, we give an explicit approximation of its distribution under extra assumption on the inactivity and activity switching rates.

3.13 Computing the untruncated signature kernel with polynomial boundary conditions

Francesco Piatti, Imperial College London

Monday 11:30-12:00

Signature kernels are becoming increasingly popular among kernel methods for sequential data. In the paper “The Signature Kernel is the solution of a Goursat PDE"", the authors prove that the untruncated signature kernel is the solution to a Goursat problem for a hyperbolic PDE with two independent time variables. Additionally, they devise a finite difference scheme to solve the PDE, which results in second-order convergence. In our study we introduce two higher-order schemes and provide theoretical guarantees for convergence. Experimental results demonstrate that our methods outperform traditional finite-difference schemes by several orders of magnitude in terms of mean absolute error, without increasing computational complexity. Similar to finite-difference methods, our schemes can also exploit GPU parallelisation to reduce the computational complexity from quadratic to linear. Finally, the practical significance of our methods is demonstrated in downstream tasks using support vector machines (SVMs) for both classification and regression problems.

3.14 Rough SDEs with jumps

Jost Pieper, University of Durham

Monday 13:30-14:00

In this talk we introduce a new version of the stochastic sewing lemma capable of dealing with discontinuous controls which may be considered as a stochastic version of the generalized sewing lemma introduced in Friz & Zhang in 2018. As an application of this we extend the theory of rough stochastic differential equations introduced by Friz, Hocquet and Lê in 2021 to allow for càdlàg rough and martingale driving noise.

3.15 Rough Paths on Affine Algebraic Varieties

Rosa Preiß, MPI MiS Leipzig

Wednesday 11:30-12:00

Bounded variation paths on an algebraic hypersurface, e.g. a sphere S^n, can be characterized by a purely algebraic condition in terms of the signature. This condition is derived from the equations of the hypersurfaces using the non-associative non-commutative halfshuffle product. Thus, it makes sense to define rough paths on the hypersurface as those satisfying the same algebraic constraints on the signature. We conjecture that this definition is equivalent to the Cass-Driver-Litterer notion of constrained rough paths in the case of a smooth hypersurface, but highlight that our algebraic definition naturally allows to treat the case of hypersurfaces with singularities in the very same terms.

3.16 Shortest-path recovery from signature with an optimal control approach

Marco Rauscher, TU Munich

Monday 10:00-10:30

In this talk, we consider the signature-to-path reconstruction problem from the control theoretic perspective. Namely, we design an optimal control problem whose solution leads to the minimal-length path that generates a given signature. In order to do that, we minimize a cost functional consisting of two competing terms, i.e., a weighted final-time cost combined with the L^2-norm squared of the controls. Moreover, we can show that, by taking the limit to infinity of the parameter that tunes the final-time cost, the problem Γ converges to the problem of finding a sub-Riemannian geodesic connecting two signatures. Finally, we provide an alternative reformulation of the latter problem, which is particularly suitable for the numerical implementation.

3.17 Probabilistic representation of nonlinear Fokker-Planck equations and nonlinear PDEs by nonlinear Markov processes

Marco Rehmeier, Bielefeld University

Monday
16:00-16:30

We introduce a notion of nonlinear Markov processes, which are naturally related to nonlinear Fokker-Planck equations, in particular to important nonlinear PDEs such as Burgers, porous media or 2D Navier-Stokes vorticity equations. We establish basic properties of these processes and show that the solutions to each of these PDEs have a probabilistic representation as the one-dimensional time marginals of a uniquely determined nonlinear Markov process. Joint work with Michael Röckner (Bielefeld).

3.18 Free generators and Hoffman's isomorphism for the two-parameter shuffle algebra

Leonard Schmitz, MPI MIS Leipzig

Wednesday
11:00-11:30

Signature transforms have recently been extended to data indexed by two and more parameters. With free Lyndon generators, ideas from B_∞ algebras and a novel two-parameter Hoffman exponential, we provide three classes of isomorphisms between the underlying two-parameter shuffle and quasi-shuffle algebras. In particular, we provide a Hopf algebraic connection to the (classical, one-parameter) shuffle algebra over the extended alphabet of connected matrix compositions. This is joint work with Nikolas Tapia.

3.19 Equilibrium Fluctuations for Weakly Asymmetric Interacting Particle Systems with Boundaries

Leander Schnee, FU Berlin

Tuesday
14:30-15:00

We consider the scaling limit of the fluctuation field around the equilibrium of a particle system with a Hamiltonian of exponential type and added noise. Scaling the time by n^2 (diffusive) and the asymmetric Hamiltonian part by $n^{-1/2}$ (weak) it is known that the equilibrium fluctuations converge to the energy solutions of the Stochastic Burgers' Equality (SBE). This result is directly related to the KPZ universality class. We investigate different ways to get a boundary condition for this limiting equation by looking at modified particle dynamics. In particular adding a heat bath to the microscopic model that moves with the asymmetric drift of the system and scaling it by $n^{-\delta}$, we show that for $\delta \leq 1$ we get the SBE equation with one Dirichlet boundary which is exhibited in different ways depending on the different values of δ. For $\delta > 1$ we see the SBE on $\mathbb{R}$ without a boundary, which is known to be well-posed. This is a joint work in progress with Ana Djurdjevac, Cédric Bernardin and Patrícia Gonçalves.

3.20 A gradient flow on control space with rough initial condition

Florin Suciu, Université Paris Dauphine-PSL

Tuesday
16:00-16:30

We consider the (sub-Riemannian type) control problem of finding a path going from an initial point x to a target point y, by only moving in certain admissible directions. We assume that the corresponding vector fields satisfy the Hörmander condition, so that the classical Chow-Rashevskii theorem guarantees the existence of such a path. One natural way to try to solve this problem is via a gradient flow on control space. However, since the corresponding dynamics may have saddle points, any convergence result must rely on suitable (e.g. random) initialization. We consider the case when this initialization is irregular, which is conveniently formulated via Lyons' rough path theory. In some simple cases, we manage to prove that the gradient flow converges to a solution, if the initial condition is the path of a Brownian motion (or rougher). The proof is based on combining ideas from Malliavin calculus with Łojasiewicz inequalities. A possible motivation for our study comes from the training of deep Residual Neural Nets, in the regime when the number of trainable parameters per layer is smaller than the dimension of the data vector.

3.21 A multiplicative surface signature through its Magnus expansion

Nikolas Tapia, WIAS Berlin

Wednesday
10:00-10:30

We present a two-parameter analog to Chen's iterated-integrals signature, which can be applied to image data. It is based on 'surface development' from higher catagory and Kapranov's construction of the analog of the free Lie algebra. Its crucial property is a two-parameter Chen's identity. This enables efficient, and parallelizable, computation of the signature. Our approach is based on the Magnus expansion, which allows us to compute, up to a certain order, explicit expressions for the integrals appearing. On the analytic side, we provide a sewing lemma for surface development, which allows to go beyond the smooth case. This talk is based on joint work with I. Chevyrev, J. Diehl and K. Ebrahimi-Fard.

3.22 Log Neural Controlled Differential Equations: The Lie Brackets Make a Difference

Benjamin Walker, University of Oxford

Monday
11:00-11:30

The vector field of a controlled differential equation (CDE) describes the relationship between a control path and the evolution of a solution path. Neural CDEs (NCDEs) treat time series data as observations from a control path, parameterise a CDE's vector field using a neural network, and use the solution path as a continuously evolving hidden state. As their formulation makes them robust to irregular sampling rates, NCDEs are a powerful approach for modelling real-world data. Building on neural rough differential equations (NRDEs), we introduce Log-NCDEs, a novel, effective, and efficient method for training NCDEs. The core component of Log-NCDEs is the Log-ODE method, a tool from the study of rough paths for approximating a CDE's solution. Log-NCDEs are shown to outperform NCDEs, NRDEs, the linear recurrent unit, S5, and MAMBA on a range of multivariate time series datasets with up to 50,000 observations.

4. Participants

Peter Bank
Christian Bayer
Florian Bechtold
Franziska Bielert
Matthew Buckland
Fabio Bugini
Oleg Butkovsky
Niels Cariou-Kotlarek
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Marco Rehmeier
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