Thursday, 14.07.2016, 09:00 - 22:00

08:00 - 09:00	REGISTRATION: WIAS Berlin, Mohrenstrasse 39, 10117 Berlin
09:00 - 09:15	OPENING
09:15 - 10:15	Chairman: Pavel Exner
09:15 - 09:40	Selfadjoint operators with boundary conditions on not closed hypersurfaces and the direct scattering problem **Andrea Mantile**
09:45 - 10:10	Limiting absorption principle, generalized eigenfunctions and scattering matrix for Laplace operators with boundary conditions on hypersurfaces **Andrea Posilicano**
10:15 - 10:45	COFFEE BREAK
10:45 - 12:15	Chairman: Igor Popov
10:45 - 11:10	Scattering Matrices for realizations of Schrödinger operators in exterior domains **Mark Malamud**
11:15 - 11:40	Renormalizations of generalized boundary triplets with applications to Laplacians **Seppo Hassi**
11:45 - 12:10	Boundary triplets for sum of tensor products of operators **Anton Boitsev**
12:15 - 14:00	LUNCH
14:00 - 15:30	Chairman: Mark Malamud
14:00 - 14:25	Singular Schrödinger operators and Robin billards: spectral properties and strong coupling expansions **Pavel Exner**
14:30 - 14:55	Leaky conical surfaces: spectral asymptotics and isoperimetric properties **Vladimir Lotoreichik**
15:00 - 15:30	Variational estimates for singular Schrödinger operators **Konstantin Pankrashkin**
15:30 - 16:00	COFFEE BREAK
16:00 - 18:00	Chairman: Konstantin Pankrashkin
16:00 - 16:25	On $\mathcal{PT}$ symmetric operators related to Hamilitonians with complex potentials **Carsten Trunk**
16:30 - 16:55	Extension theory models and resonance states completeness **Igor Popov**
17:00 - 17:25	Photon generation in quantum graphs with time dependent boundary conditions **Alexander Trifanov**
17:30 - 17:55	Quantum graph model of quantum fields with closed timelike curves **Igor Lobanov**
19:00 - 22:00	DINNER

Friday, 15.07.2016, 09:00 - 18:00

09:00-10:30	Chairman: Valentin Zagrebnov
09:00 - 09:25	Unique continuation principle and its absence on continuum space, on lattices and on quantum graphs **Ivan Veselic**
09:30 - 09:55	Inverse problems for quantum graphs - a boundary triples perspective **Jonathan Rohleder**
10:00 - 10:25	The Airy equation on a quantum graph **Delio Mugnolo**
10:30 - 11:00	COFFEE BREAK
11:00-12:00	Chairman: Ivan Veselic
11:00 - 11:25	Construction of dynamical semigroups by regularisation à la Kato **Valentin Zagrebnov**
11:30 - 11:55	On operator-norm estimates for approximations of solutions of evolution equations using the Trotter product formula **Artur Stephan**
12:00 - 14:00	PHOTO + LUNCH
14:00 - 15:30	Chairman: Volodymyr Derkach
14:00 - 14:25	Squeezing of arbitrary order **Franciszek H. Szafraniec**
14:30 - 14:55	Zeros of negative type and finite rank perturbations **Henrik Winkler**
15:00 - 15:25	On the spectrum of J-frame operators **Francisco Martínez Pería**
15:30 - 16:00	COFFEE BREAK
16:00 - 17:05	Chairman: Franciszek H. Szafraniec
16:00 - 16:25	On a class of generalized Stieltjes continued fractions **Volodymyr Derkach**
16:30 - 16:55	Weighted Procrustes problems **Juan Ignacio Giribet**
17:00 - 17:05	CLOSING

Boundary triplets for sum of tensor products of operators

Anton Boitsev
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In this talk we consider an operator having form $S = A \otimes I + I \otimes T$, where A is symmetric and T is self adjoint. Our aim is to obtain a boundary triplet and, finally, the Weyl function, corresponding to S^*. It turns out that the case when T is bounded is much easier. However, we consider both cases and obtain all the formulas in terms of the boundary triplet and the Weyl function, corresponding to A^*. Moreover, we demonstrate some applications of obtained technique to physical problems.

On a class of generalized Stieltjes continued fractions

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With each sequence of real numbers two kinds of continued fractions are associated - the so-called P-fraction and a generalized Stieltjes fraction, which in the case when this sequence consists of moments of a probability measure on the half-line, coincide with the J-fraction and the Stieltjes fraction, respectively. A subclass of regular sequences is specified for which explicit formulas connecting these two continued fractions are found. This allows to give a description of spectral functions of indefinite strings, which can be treated as massless threads with indefinite point masses and multipoles.

Singular Schrödinger operators and Robin billards: spectral properties and strong coupling expansions

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In this talk we discuss spectral properties and strong coupling expansions of several operator classes. They include Schrödinger operators with an attractive singular "potential", supported by a manifold of a lower dimensionality. The simplest of them of them can be formally written as $-\Delta - \alpha\delta(x - \Gamma)$ with $\alpha > 0$, where Γ is a curve in $\mathbb{R}^d$; $d = 2; 3$, or a surface in $\mathbb{R}^3$; the expression can be modified to include a different singular interaction term or a regular potential bias. Another class are Hamiltonians describing quantum motion in a region with attractive Robin boundary. We discuss the ways in which spectral properties of such systems are influenced by the interaction support geometry with an attention to similarities and differences between the operators considered.

Weighted Procrustes problems

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Let $\mathcal{H}$ be a Hilbert space, $L(\mathcal{H})$ the algebra of bounded linear operators on $\mathcal{H}$ and $W \in L(\mathcal{H})$ a positive operator such that $W^{1/2}$ is in the p-Schatten class, for some $1 \leq p < \infty$. Given $A \in L(\mathcal{H})$ with closed range and $B \in L(\mathcal{H})$, we study the following weighted approximation problem: analize the existence of

$$\min_{X \in L(\mathcal{H})} \|AX - BW\|_p, \qquad \text{where} \qquad X = W^{1/2} X_p.$$

In this talk we show that the existence of this minimum is equivalent to a compatibility condition between $R(B)$ and $R(A)$ involving the weight W, and we characterize the operators which minimize this problem as W-inverses of A in $R(B)$.

Renormalizations of generalized boundary triplets with applications to Laplacians

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In this talk abstract boundary mapping techniques combined with analytic methods based on the notion of associated Weyl functions are discussed for studying operator and spectral theoretic as well as geometric properties of selfadjoint extensions of symmetric operators. The main aim is to introduce some renormalization techniques for general classes of boundary triplets. We also treat applications for Laplacian operator on bounded domains with smooth, Lipschitz, or rough boundary. The talk is based on some joint work with Vladimir Derkach and Mark Malamud.

Quantum graph model of quantum field with closed timelike curves

Igor Lobanov
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In 1991 David Deutsch demonstrated that time-travel paradoxes can be avoided using density matrix formalism. The Deutsch model deals with distinguishable particles, but since than the model was generalized to cover quantum fields, e.g. by Pienaar, Myers and Ralph. On this basis we build a theory of quantum field on quantum graph with closed timelike curves. The theory incorporates the theory of optical quantum circuits and can be used to help plan experiments simulating time travel.

Leaky conical surfaces: spectral asymptotics and isoperimetric properties

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We will consider three-dimensional Schrödinger operator with attractive δ-interaction of constant strength supported on the unbounded circular conical surface. This Hamiltonian can be defined as a self-adjoint operator either via its quadratic form or as a self-adjoint extension of an underlying symmetric operator. In quantum mechanics it models a charged particle attracted by the (leaky) conical surface.

We will characterise the essential and discrete spectra of this operator and present recent results on the asymptotics of its eigenvalue counting function. We intend also to discuss spectral properties of Schrödinger operators with δ-interactions supported on more general (not necessarily circular) conical surfaces. We will show that under reasonable constraints the circular conical surface maximises the lowest eigenvalue.

The main motivation to study spectra of such Hamiltonians comes from spectral geometry. The ultimate goal is to understand spectral properties of δ-interactions supported on arbitrary asymptotically flat surfaces.

The results in this talk are obtained in collaboration with J. Behrndt, P. Exner, and T. Ourmières-Bonafos.

Scattering matrices for realizations of Schrödinger operators in exterior domains

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Let A be a symmetric operator in a Hilbert space $\mathfrak{H}$ with infinite deficiency indices $n_{\pm}(A) = \infty$. We investigate the scattering matrix of two extensions A_0 and A_1 assuming that they are resolvent comparable, i.e. their resolvent difference is of trace class. The scattering matrix is expressed by means of the limit values of the abstract Weyl function. The latter is defined in the framework of double B-generalized boundary triples demonstrating an interesting feature of our approach.

The abstract result is applied to different realizations of Schrodinger differential expressions in exterior domains in $\mathbb{R}^2$. In particular, if A_D and A_N are the Dirichlet and Neumann realizations, then the scattering matrix of the scattering system $\{A_D, A_N\}$ is expressed by means of the limit values of the Dirichlet-to-Neumann map. The latter is the classical object naturally appeared in the theory of boundary value problems of the second order elliptic operators.

The talk is based on joint results with J. Behrndt and H. Neihardt.

Selfadjoint operators with boundary conditions on not closed hypersurfaces and the direct scattering problem

Andrea Mantile
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The abstract theory of self-adjoint extensions of symmetric operators is used to build self-adjoint realizations of Schrödinger-type operators with linear boundary/interface conditions on (a relatively open part of) a compact hypersurface in $\mathbb{R}^n$. Our approach allows to obtain Krein-like resolvent formulae where the reference operator coincides with the free operator corresponding to our model, i.e.: the Schrödinger operator defined on regular functions without interface conditions. This provides an useful tool for the scattering problem from a hypersurface. Schatten-von Neumann estimates for the difference of the powers of resolvents (of the free and the perturbed models) yield the existence and completeness of the wave operators of the associated scattering systems, while a limiting absorption principle for singular perturbations allows us to obtain the generalized eigenfunctions and the scattering matrix; both these objects are written in terms of operator-valued Weyl functions.

This is a joint work with A. Posilicano and M. Sini

The Airy equation on a quantum graph

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The Airy equation is a one-dimensional partial differential equation of first order in time and third order in space; it is mostly relevant as the linear part of the classical KdV equation. An explicit formula for the Airy equation on the line is known and shows that the solution is given by a unitary group with dispersive features. Little is known about different geometrical settings, but unitary group generation cannot be expected on bounded and semibounded intervals. I am going to show how the theory of Krein spaces can be used to fully characterize self-adjoint extensions on a star graph.

This is joint work with Diego Noja (Milano) and Christian Seifert (Hamburg/München).

Variational estimates for singular Schrödinger operators

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We are presenting some variational estimates for the eigenvalues of Schrödinger operators with singular interactions whose support is non-smooth.

On the spectrum of J-frame operators

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Recently, J-frame where introduced in order to extend Hilbert space frame theory to Krein spaces. Given a Krein space $\mathcal{H}$, a J-frame is a family of vectors $\mathcal{F} = f_{ii} \in I$, composed of positive and negative vectors, which spans a pair of maximal uniformly definite subspaces. For each J-frame $\mathcal{F}$ there is an associated operator $S : \mathcal{H} \to \mathcal{H}$, selfadjoint with respect to the Krein space structure, which allows to reconstruct any vector of the Krein space as a (possibly infinite) linear combination of the vectors in $\mathcal{F}$. Along this talk, we present a spectral description of J-frame operators.

It is based in a joint work with Juan I. Giribet, M. Langer, L. Leben, A. Maestripieri, and C. Trunk.

Extension theory models and resonance states completeness

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We deal with a few models based on the operator extensions theory. The first one is a resonator with point-like boundary window (model of the Helmholtz resonator). Lax-Phillips approach is used. Resonances (quasi-eigenvalues) are eigenvalues of some dissipated operator. The corresponding resonant states belong to $L_2(\Omega)$ for any bounded Ω. An interesting quaestion appeared: What is the maximal domain Ω such that the resonant states are complete in $L_2(\Omega)$? It is proved that for convex bounded resonator Ω^{in} in $\mathbb{R}^3$, the set of the resonant states of the Laplacian forms a basis in $L_2(\Omega^{in})$. A relation with Sz.-Nagy functional model is analyzed.

Limiting absorption principle, generalized eigenfunctions and scattering matrix for Laplace operators with boundary conditions on hypersurfaces

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We provide a limiting absorption principle for the self-adjoint realizations of Laplace operators corresponding to boundary conditions on (relatively open parts Σ of) compact hypersurfaces $\Gamma = \partial\Omega$, $\Omega \subset \mathbb{R}^n$. For any of such self-adjoint operators we also provide the generalized eigenfunctions and the scattering matrix; both these objects are written in terms of operator-valued Weyl functions. We make use of a Krein-type formula which provides the resolvent difference between the operator corresponding to self-adjoint boundary conditions on the hypersurface and the free Laplacian on the whole space $\mathbb{R}^n$. Our results apply to standard examples of boundary conditions, like Dirichlet, Neumann, Robin, δ and δ'-type, either assigned on Γ or on $\Sigma \subset \Gamma$.

Inverse problems for quantum graphs - a boundary triples perspective

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The question whether a Schrödinger operator on a metric graph is uniquely determined by a corresponding Titchmarsh-Weyl function (acting on a part of the vertex set) is discussed from the viewpoint of boundary triples. Several choices of Titchmarsh-Weyl functions will be considered. A particular focus will be on the influence of geometric properties of the underlying graph to the uniqueness question in the inverse problem.

On operator-norm estimates for approximations of solutions of evolution equations using the Trotter product formula

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We approximate the solution of a non-autonomous linear evolution equation in the operator-norm topology. The solution method based on an extension and perturbation problem for linear operators. The approximation is derived using the Trotter product formula. As an example we consider the diffusion equation perturbed by a time dependent potential.

Squeezing of arbitrary order

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I intend to discuss the matters by means of extension theory and orthogonal polynomials. Talk based on the paper

Katarzyna Górska, Andrzej Horzela and FHSz. Squeezing of arbitrary order: the ups and downs. *Proc. R. Soc. Lond. Ser. A Math. Phys. Eng. Sci.* 470 (2014), no. 2172, 20140205, 21 pp.

Photon generation in quantum graphs with time dependent boundary conditions

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The behavior of quantum particle in a net structure is of primary interest from both theoretical and experimental point of view. Due to highly importance of this problem for quantum devices implementation numerous works were devoted to spectral and transport problems in quantum waveguides. One of the most productive abstractions of real quantum network of low dimensions is quantum graph. We referred to photonic transport in optical waveguide structures. Due to weak interaction between photons in linear medium the problem of quantum field evolution doesn't seem to be very difficult many-body problem and can allow some beautiful descriptions in quantum graph theory. One of such a problems is photon generation in a domain with moving boundary due to nonadiabatic interaction between internal modes. Actually, in experiments this process has very low efficiency due to requirements to boundary motion: typical frequency of oscillations should be of the order of radiation frequency, which make impossible to realize the motion using mechanical systems. Nevertheless such a systems may be constructed in superconductive devices to produce microwave photons. We suggest a model of photon generator in the waveguide structure which efficiency depends on network geometry.

On $\mathcal{PT}$ symmetric operators related to Hamilitonians with complex potentials

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Consider $\mathcal{PT}$ symmetric Hamiltonians

$$(\tau y)(x) := -y''(x) + x^2(ix)^\epsilon y(x), \quad \epsilon > 0.$$

where ϵ is a real number not smaller than 2, the eigenvalue problem is defined using a contour in the complex plane (not in $\mathbb{R}$). It is assumed that this contour is between two so-called Stokes lines.

In the Sturm-Liouville theory for Hamiltonians with a complex potential there exists a limit point/limit circle classification which gives a mathematical interpretation to the Stokes lines (and Stokes wedges).

Moreover, we identify a setting where the corresponding Hamiltonian turns out to be selfadjoint in the Krein space $(L_2(\mathbb{R}), [\cdot, \cdot])$. The corresponding Krein space inner product is defined with the parity $\mathcal{P}$ as the Gramian. Finally, we present some results on the location of the resolvent set.

Unique continuation principle and its absence on continuum space, on lattices and on quantum graphs

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A powerful tool in the analysis of solutions of partial differential equations are unique continuation principles. Quantitative versions play an important role in inverse problems, uniqueness theorems for linear and non-linear differential equations, and in the theory of random Schroedinger operators. On the contrary quantum graphs violate the continuation principle, giving rise to new phenomena. Certain graph Laplacians exhibit similar features.

Zeros of negative type and finite rank perturbations

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A generalized Nevanlinna function Q with one negative square has precisely one generalized zero of negative type in the closed upper half-plane. If the operator associated with Q undergoes a rank-one perturbation parametrized by $\tau \in R$, the global behavior of the generalized zero of negative type $\alpha(\tau)$ is studied. In particular, it is shown that it is continuous, and the local behavior in the points where the function leaves the real line is investigated.

The talk is based on a joint work with Henk de Snoo and Michal Wojtylak.

Construction of dynamical semigroups by regularisation à la Kato

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A functional version of the Kato one-parametric regularisation for construction of dynamical semigroup generators for the relative bound-one perturbations is proposed. It is illustrated by an example of the boson-number cut-off regularisation.

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