MTA SZTAKI

Planning and Scheduling in the Digital Factory

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Why "digital"?

1. Using Excel is enough?
2. Distinctive features:
   - Use real-time data
   - Complex resource models (Human workforce, raw material supply, special tooling, etc.)
   - Optimization
3. Need for
   - Mathematical models
   - Optimization algorithms
   - Visualization of results
4. Additional crucial components
   - Manufacturing Execution System with real-time feedback
   - Tracking of raw material, semi-finished goods, final products (RFID)

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Problem areas: Planning

Planning for "one-of-a-kind" products

- Each customer order becomes a project
- Projects can be long or short, but unique
- Optimization objectives: meet due-dates, minimize resource hiring costs

Problem areas: Scheduling

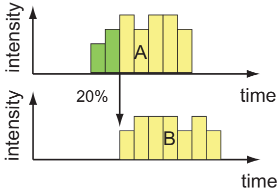
I. Scheduling with several routing alternatives
   - Jobs have several routing alternatives
   - One routing alternative must be chosen for each job
   - The jobs must be sequenced on the selected machines
   - Optimization criterion: schedule length

II. Resource leveling
   - Beside machines, there are resources needed by the tasks (workforce, energy)
   - Each resource has a desired (maximum) usage level, and a function measuring the difference between the desired level and the actual utilization
   - The tasks are assigned to machines, and have some time windows
   - Objective: find a schedule of the tasks which minimizes the difference between the desired and actual resource usage.

Part I: Planning for "one-of-a-kind" products

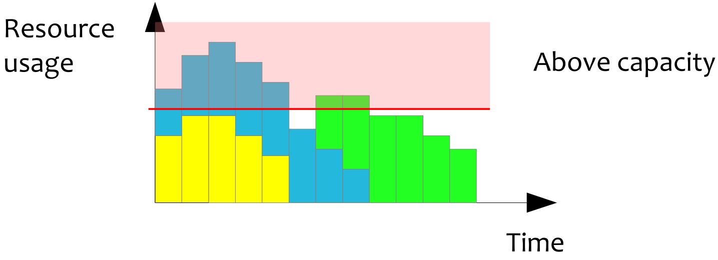
Planning for "one-of-a-kind" products

1. Each customer order is a project.
2. Projects consist of activities like 'Design', 'Process planning', 'Manufacturing', 'Assembly', 'Testing', etc.
3. Activities require various renewable resources like designers, machine tools, workforce with appropriate skills, etc.
4. The intensity of each activity may vary over time.
5. Activities have time windows in which they must be completed.
6. Feeding precedence constraints between pairs of activities:



Planning objective

As each resource (of the manufacturer) has a finite capacity, and the activities have time windows, to find feasible solutions extra resources may have to be hired/subcontracted.



Mathematical modeling

1. Problem data
   - Finite time horizon divided into time periods $\mathcal{T} = \{[0,1],[1,2],\ldots,[T-1,T]\}$.
   - Resources $R_1,\ldots,R_k,\ldots R_m$ with capacities b_t^k, $t \in \mathcal{T}$.
   - Activities $A_1,\ldots,A_n$, with parameters: length p_i, time windows $[r_i,d_i]$, max intensity a_i, resource requirements q_i^k
2. Decision variables:

$$z_t^{i,f} = \begin{cases} 1 & \text{if less than an } f \text{ fraction of activity } A_i \text{ is processed} \\ & \text{up to time } t, \\ 0 & \text{otherwise.} \end{cases}$$

$$x_t^i = \text{ amount of work planned in time period [t-1,t)}$$

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The MIP formulation

$$\min \sum_{k\in R} \sum_{t=1}^{T} c_t^k y_t^k$$

subject to

$$\begin{aligned}
\sum_{t=r^i}^{d^i} x_t^i &= 1, && i \in N, \\
\sum_{t=r^i}^{\ell-1} x_t^i &\geq f(1 - z_\ell^{if}), && i \in N,\ f \in F^i, \\
& && \ell \in \{r^i + p^{if}, \ldots, d^i\}, \\
x_t^j &\leq a^j(1 - z_t^{if}), && i \in N,\ (i,j,f) \in E^i, \\
z_t^{if} &\geq z_{t+1}^{if}, && i \in N,\ f \in F^i, \\
& && t \in \{r^i + p^{if}, \ldots, d^i - 1\},
\end{aligned}$$

The MIP formulation

$$\sum_{t=r^i}^{\ell} x_t^i \geq \sum_{t=r^j}^{\ell} x_t^j, \qquad (i,j,f) \in E^i, \; \ell \in \{\max\{r^i, r^j\}, \ldots, \min\{d^i, d^j\}\},$$

$$\sum_{i \in N_t^k} q_k^i \cdot x_t^i \leq b_t^k + y_t^k, \qquad k \in R, t \in \{1, \ldots, T\},$$

$$0 \leq x_t^i \leq a^i, \qquad i \in N, \; t \in \{r^i, \ldots, d^i\},$$

$$0 \leq y_t^k \leq \bar{b}_t^k, \qquad k \in R, \; t \in \{1, \ldots, T\},$$

$$z_t^{if} \in \{0,1\}, \qquad i \in N, \; f \in F^i, \; t \in \{r^i + p^{if}, \ldots, d^i\},$$

How to solve it?

- Exact solution approaches (always deliver optimal solutions)
  - Branch-and-Price
    E. W. Hans (2001), Resource loading by branch-and-price techniques, Ph.D. thesis, Twente University Press
  - Branch-and-Cut
    T. Kis (2006). RCPS with variable intensity activities and feeding precedence constraints, In: J. Józefowska, and J. Wegalrz, Perspectives in Modern Project Scheduling, Springer, New York, 2006, pp. 105-129.
- Heuristics
  - Based on solving linear relaxations
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New valid inequalities from precedence constraints

Precedence constraint: %Completed-to-Start

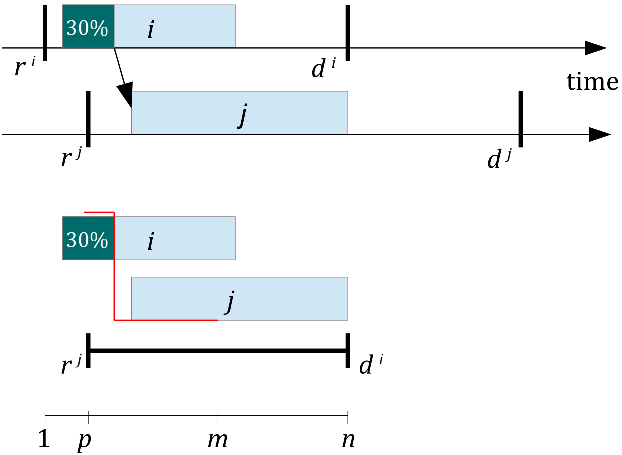
$$\sum_{t=r^i}^{d^i} x_t^i = 1, \qquad i \in N, \tag{1}$$

$$\sum_{t=r^i}^{\ell-1} x_t^i \geq f(1 - z_\ell^{if}), \qquad i \in N,\ f \in F^i,\ \ell \in \{r^i + p^{if}, \ldots, d^i\}, \tag{2}$$

$$x_t^j \leq a^j(1 - z_t^{if}), \qquad i \in N,\ (i,j,f) \in E^i, \tag{3}$$

$$z_t^{if} \geq z_{t+1}^{if}, \qquad i \in N,\ f \in F^i,\ t \in \{r^i + p^{if}, \ldots, d^i - 1\}, \tag{4}$$

Illustration



Derivation of valid inequalities

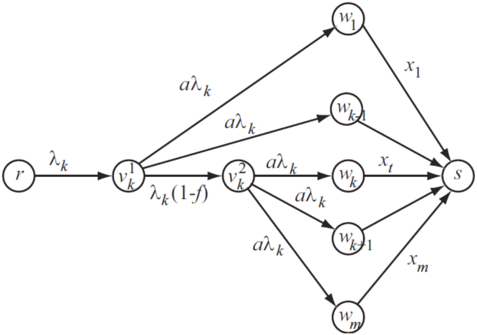
$K_f = \{(x,z) \in \mathbb{R}^n \times \{0,1\}^{m-p+1} \mid (x,z)$ satsifies the system

$$
\begin{aligned}
\sum_{t=1}^{n} x_t &= 1, \\
\sum_{t=1}^{\ell-1} x_t &\geq f \cdot (1 - z_\ell), \quad \ell \in \{p+1, \ldots, m\}, \\
\sum_{t=1}^{m} x_t &\geq f, \\
z_t &\geq z_{t+1}, \quad t \in \{p+1, \ldots, m-1\}, \\
0 \leq x_t &\leq a, \quad t \in \{1, \ldots, n\}.
\end{aligned}
$$

$\}$

Network representation to derive valid inequalities

for $k = p, \dots, m$ consider the set of edges between the vertices $\{r, s\} \cup \{v_p^1, v_p^2, \dots, v_m^1, v_m^2\} \cup \{w_1, \dots, w_m\}$:



where $\lambda_k = \begin{cases} 1 - z_{p+1} & \text{if } k = p, \\ z_k - z_{k+1}, & \text{if } k \in \{p+1, \dots, m-1\}, \\ z_m, & \text{if } k = m. \end{cases}$

Max-flow min-cut yields new valid inequalties

Theorem

K_f equals the set of vectors (x, z) in $[0, a]^n \times [0, 1]^{m-p}$ that satisfy the following linear constraints:

$$
\begin{aligned}
\sum_{t=1}^{n} x_t &= 1 \\
a_r z_{t_1} + a \sum_{t \in S_1 \setminus \{t_1\}} z_t + \sum_{t \in \{1,\ldots,t_1\} \setminus (S_1 \cup S_2)} x_t &\geq f - a|S_2| \\
(f-1) z_\ell + \sum_{t \in U_\ell^1} a z_t + \sum_{t \in U_\ell^2} a z_\ell + \sum_{t \in \{1,\ldots,n\} \setminus (U_\ell^1 \cup U_\ell^2)} x_t &\geq f \\
\sum_{t \in U} (x_t - a z_t) &\geq 1 - f \\
z_t - z_{t+1} &\geq 0
\end{aligned}
$$

The merits of strenghtening the formulation

1. If there can be no overlap between predecessor and successor activities ($f = 1.0$), then there is a considerable gain in solution time and quality when using the new valid inequalities along with Gomory mixed integer cuts and Flow cover inequalities.

   T. Kis (2005), A branch-and-cut algorithm for scheduling of projects with variable-intensity activities, Mathematical Programming, Vol. 103, pp. 515-539.

2. As the overlap increases (f decreases) the new inequalities become less important, which shows that the problem becomes easier.

   T. Kis (2006), RCPS with variable intensity activities and feeding precedence constraints, In: J. Józefowska, and J. Wegalrz, Perspectives in Modern Project Scheduling, Springer, New York, 2006, pp. 105-129.

Application areas

- Make-to-order manufacturing
  Each customer order becomes a project, e.g., production of machining/assembly lines
- Order acceptance
  - A reliable due date is to be determined for each **new** project.
  - The rough-cut capacity plan estimates costs, and helps to determine due dates. It is used to settle a contract with the customer, which is executable for the company.
- Re-planning in case of disturbances
  - In case of changes of production capacities (planned, or unexpected), project plans have to be adjusted, and exceptionally due-dates have to be modified.
  - Rough-cut capacity planning helps to adjust project plans in a changing manufacturing environment.
  - An important issue is that re-planning must be conservative, i.e., modify to the least extent.

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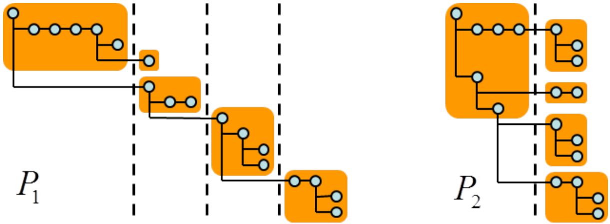
How to define projects?

Project activities can be defined in two fundamental ways

1. Human planner determines them based on past experience
2. Aggregation from detailed manufacturing process plans



J. Váncza, T. Kis, A. Kovács (2004), Aggregation – the key to integrating production planning and scheduling, CIRP Annals of Manufacturing Technology, Vol. 53, pp. 377–380.

Extensions

The basic model can be extended by new constraints:

1. Further variants of feeding precedence constraints
   - %Completed-to-Start
   - Start-to-%Completed
   - %Completed-to-Finish
   - Finish-to-%Completed

   A. Alfieri, T. Tolio, M. Urgo (2012), A project scheduling approach to production and material requirement planning in Manufacturing-to-Order environments, J. Intell. Manuf, Vol. 23, pp. 575–585
2. Non-renewable resources
   - Examples: raw-materials, energy, money, etc.
   - Sophisticated modeling of resources may be needed, e.g., energy consumption

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Part II: Scheduling of complex job shops

Scheduling of complex job shops

Problem characteristics

1. Hundreds or thousands of jobs, each with a release-date and due-date
2. Each job has alternative routings (alternative operation sequences)
3. Operations of the jobs may have machine alternatives
4. Additional resources like skilled workforce must be scheduled beside machines
5. There can be raw material, and energy consumption constraints
6. The availability of workforce and machines may vary in time (holidays, work-shifts, etc.)
7. Various objectives, like delivery accuracy, machine utilization, leveled use of workforce may have to be optimized (multi-criteria optimization)

Application I: Scheduling of lamp factory

1. Customer orders: thousands of identical products
   - Big customer orders must be divided into batches, which is an extra difficulty
2. Different products require similar technological steps
3. Production resources:
   - machines capable of performing one to several technological steps
   - Skilled workforce (machine operators)
   - raw materials
4. Each product may have a number of production alternatives (alternative routings)
5. Optimization objective:
   $\min(\text{Customer order tardiness}, \text{Total machine setup time})$
6. Industries with similar characteristics: consumer electronics (e.g., smart-phones), furniture, etc.

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Application II: Scheduling of pneumatic equipment factory

1. Customer orders: tens of identical products
2. Different products may require completely different technological steps
3. Production resources:
   - machines capable of performing one to several technological steps
   - operators may be moved between production cells, and work in shifts
   - raw materials
4. Each technological step (job operation) may have machine alternatives
5. Optimization objective: $\min(\text{Customer order tardiness}, \text{Total machine setup time})$

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Application III: Scheduling of a CNC workshop

1. Production orders: jobs with several routing alternatives each consisting of 20-30 machining operations
2. Different products may require completely different technological steps
3. Production resources:
   - machines capable of performing a subset of the machining operations
4. Each machining operation may have several machine alternatives from which exactly one must be chosen
5. Optimization objective: $\min(\text{Schedule length})$

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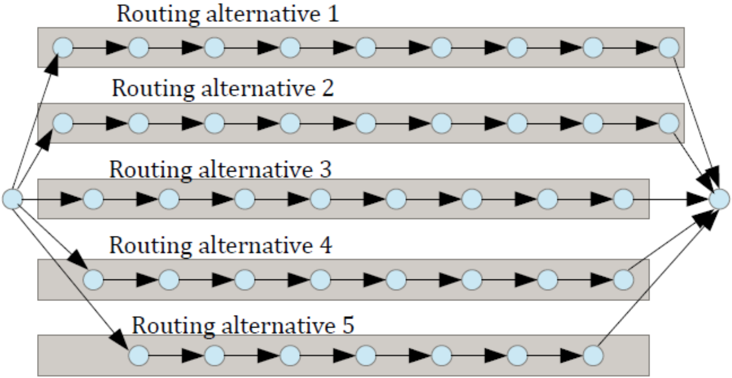
Common subproblems, and algorithms

1. Selection of routing alternatives
   - Objective: minimize maximum machine load
   - Method: Column generation & integer programming
2. Machine selection for machining operations and operation sequencing
   - Objective: multiple criteria (Customer order tardiness, machine utilization)
   - Methods: Meta-heuristics, Constraint-based scheduling
3. Resource leveling
   - Objective: leveled use of resources (according to some function)
   - Method: Proprietary approach, or integer programming

Selection of routing alternatives

Routing alternative:

1. Consists of a sequence of machining operations
2. For each machining operation one or several machines may be specified from which exactly one must be chosen



Modeling by IP with millions of columns

Objective: select a routing for each job to minimize maximum machine load

$$\text{Load Balancing} = \min L$$

such that

$$\sum_{\omega \in \Omega_j} x^j_\omega = 1, \quad \text{for each job } j$$

$$\sum_j \sum_{\omega \in \Omega_j} p^j_\omega(i) x^j_\omega \leq L, \quad \text{for each machine } i$$

$$x^j_\omega \in \{0,1\}, \quad \text{for each job } j \text{ and routing } \omega \in \Omega_j$$

where

$$p^j_\omega(i) = \left\{ \begin{array}{l} \text{total processing time of those operations assigned to} \\ \text{machine } i \text{ in routing } \omega \end{array} \right\}$$

Solve LP relaxation of Load Balancing IP by Column Generation

Column Generation: standard technique to solve LPs with many columns.

1. Start with a small subset of feasible columns
2. Iteratively add new columns until the relaxed LP admits an optimal solution for the "big" LP
3. adding new columns: pricing problem
   - use LP duality to find new columns

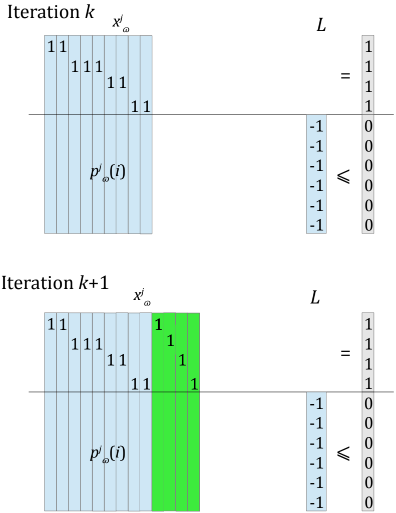
How does it work for the LP relaxation of our Load Balancing IP?

- Pricing problem:

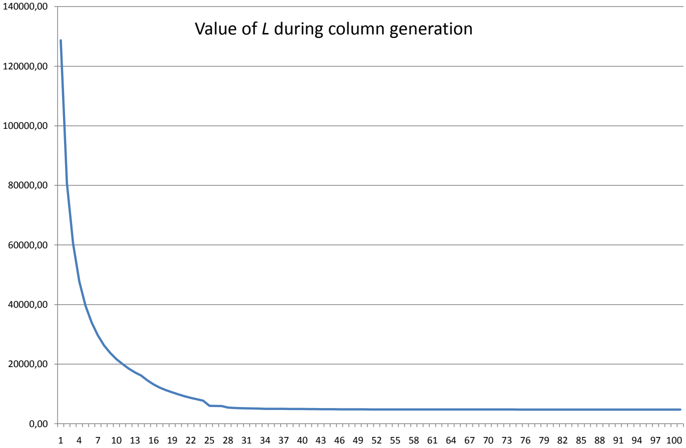
$$\min\{\bar{v}_j - \sum_{i \in \text{Machines}} p^j_\omega(i)\bar{w}_i \mid \omega \in \Omega_j\}$$

- If the optimum value of the pricing problem is negative, a new column must be added to LP

Illustration



Graph of the objective function value over 100 iterations

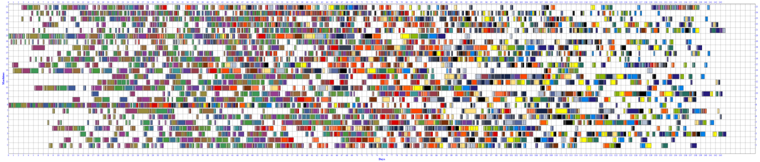


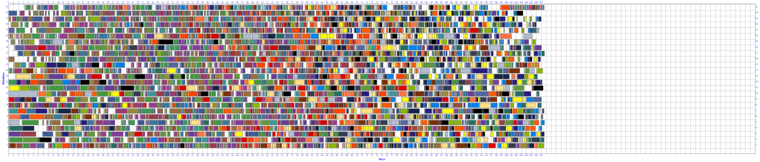
How the results of Load Balancing can be used?

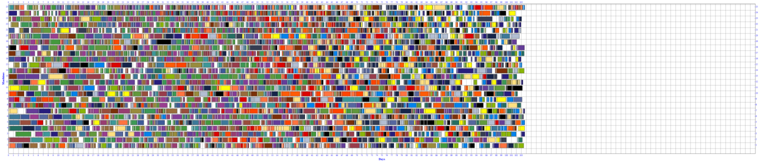
The complete procedure for scheduling jobs with routing alternatives

1. Solve the LP relaxation of Load Balancing IP by column generation
   - Output: a set of columns, usually several routings for the same job
2. On the set of columns, solve the restricted MIP
   - This gives an upper bound on the true optimum, but typically very close ($\sim 4\%$).
3. Make an initial schedule using the routings of the jobs
4. Improve the initial schedule by local search based heuristic method, or by constraint programming based method
   - At this stage the routing alternatives are fixed, but new machines may be assigned to the operations of the selected routing alternatives.

Gantt charts of schedules at the various stages







The impact of proper load balancing

Comparison of HLB and OLB

Average $(I_{OLB} - C_{HLB})/C_{HLB}$	3 %
Average $(C_{HLB} - C_{OLB})/C_{OLB}$	23 %
Average $(I_{HLB} - C_{HLB})/C_{HLB}$	51 %
Average $(I_{OLB} - C_{OLB})/C_{OLB}$	26 %

where

- HLB = heuristic load balancing
- OLB = optimal load balancing (the method just presented)
- I_{HLB} = initial schedule length after HLB
- I_{OLB} = initial schedule length after OLB
- C_{HLB} = final schedule length after HLB
- C_{OLB} = final schedule length after OLB

Resource leveling

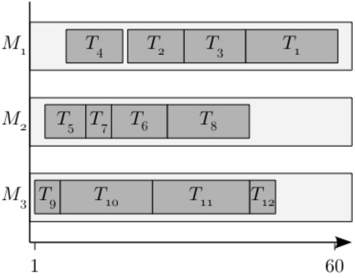
Definition

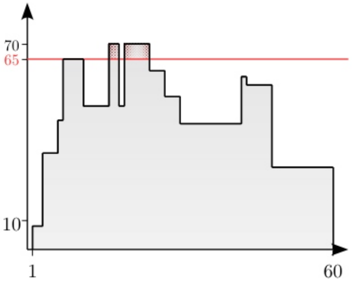
Resource profile of schedule $\mathcal{S}$
Vector function $R^{\mathcal{S}} = (r_1^{\mathcal{S}}, \ldots, r_L^{\mathcal{S}})$, where $r_\ell^{\mathcal{S}} : [0, D+1] \to \mathbb{Q}_+$ gives the total requirement from resource ℓ at time t in schedule $\mathcal{S}$, i.e.,

$$r_\ell^{\mathcal{S}}(t) := \sum_{j: S_j < t \leq S_j + p_j} b_\ell^j,$$

where the summation is over all tasks running at time t, and b_ℓ^j is the usage of task j from resource ℓ.

Illustration





Objective functions

Definition

Measures of the leveled use of resources

$$f(R^{\mathcal{S}}) := \sum_{\ell=1}^{L} \sum_{t=0}^{D} \hat{f}_\ell(r_\ell^{\mathcal{S}}(t), C_\ell)$$

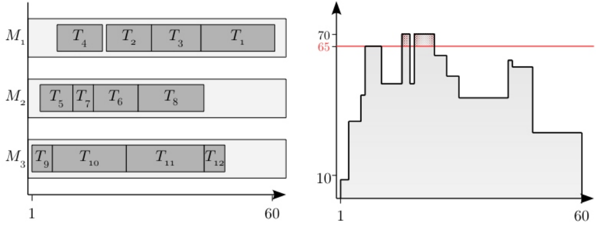
where $\hat{f}_\ell : \mathbb{Q} \times \mathbb{Q} \to \mathbb{Q}_+$ satisfies $\hat{f}_\ell(x, y - z) = \hat{f}_\ell(x + z, y)$, and C_ℓ is the desired usage level for resource ℓ.

Examples:

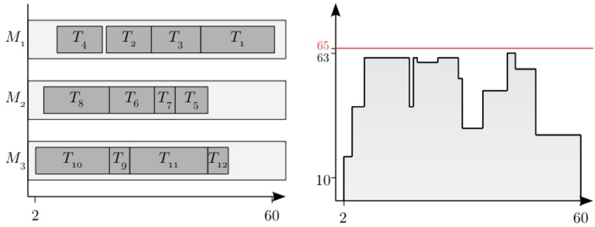
- Linear: $\hat{f}_\ell(x, y) = w_\ell \max\{0, x - y\}$
- Quadratic: $\hat{f}_\ell(x, y) = w_\ell(x - y)^2$

Reshuffling of tasks to achieve leveled resource usage

Schedule $\mathcal{S}_1$



Schedule $\mathcal{S}_2$



Problem data

1. Machines
2. Tasks: processing time p_j, time window $[e_j, d_j]$, dedicated to machine m_j
3. Resources: functions $\hat{f}_\ell(x, y)$, desired level C_ℓ
4. Schedules: each task starts and ends in its time window, and task on the same machine do not overlap in time
5. Objective: find a schedule $\mathcal{S}$ which minimizes $f(R^{\mathcal{S}})$

Modeling by mixed-integer program

$$\min \sum_{\ell=1}^{L} \sum_{t=0}^{D} \hat{f}_\ell(y_{\ell t}, C_\ell)$$

subject to

$$\begin{aligned}
\sum_{t \in \{e_j, \dots, d_j - p_j\}} x_{jt} &= 1, \quad \forall\, j \in N, \\
\sum_{j \in N_i} \sum_{\tau = t - p_j + 1}^{t} x_{j\tau} &\leq 1, \quad \forall\, t \in \{0, \dots, D\},\ i \in \{1, \dots, m\} \\
\sum_{j \in N} \sum_{\tau = t - p_j + 1}^{t} b_\ell^j x_{j\tau} - y_{\ell t} &= 0, \quad \forall\, t \in \{0, \dots, D\},\ \ell \in \{1, \dots, L\} \\
x_{jt} &\in \{0, 1\}, \quad \forall j \in N,\ t \in \{e_j, \dots, d_j - p_j\}.
\end{aligned}$$

Complexity

Theorem

The resource leveling problem is NP-hard both for the linear and the quadratic objective function, even if there is only a single resource, the schedule of all but one machines is fixed, and on the remaining machine the order of operations can be changed.

Theorem

The resource leveling problem is solvable in polynomial time both for the linear, and the quadratic objective function, provided that the schedule of all but one machines is fixed, and the ordering of operations on the remaining machines is fixed, but the starting time of operation on that machine can be changed.

Drótos M., and Kis, T. (2011), Resource leveling in a machine environment, European J. Operational Research, Vol. 212, 12-21.

Lagrange relaxation

$$LB(\boldsymbol{\lambda}) = \sum_{i=1}^{m} LB_i(\boldsymbol{\lambda}) + \min_{y} \sum_{\ell=1}^{L} \sum_{t=0}^{D} \left(\hat{f}_\ell(y_{\ell t}, C_\ell) - \lambda_{\ell t} y_{\ell t} \right),$$

where $LB_i(\boldsymbol{\lambda}) = \min \sum_{\ell=1}^{L} \sum_{t=0}^{D} \sum_{j \in N_i} \sum_{\tau=t-p_j+1}^{t} \lambda_{\ell t} b_{j\ell} x_{j\tau}$

subject to $\sum_{t \in \{e_j, \ldots, d_j - p_j\}} x_{jt} = 1, \quad \forall\, j \in N_i,$

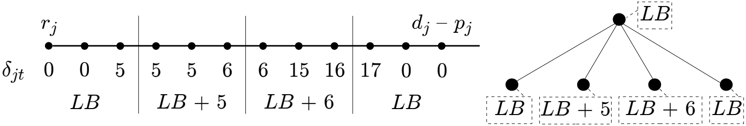
$$\sum_{j \in N_i} \sum_{\tau=t-p_j+1}^{t} x_{j\tau} \leq 1, \quad \forall\, t \in \{0, \ldots, D\}$$

$$x_{jt} \in \{0, 1\}, \quad \forall j \in N_i,\ t \in \{e_j, \ldots, d_j - p_j\}.$$

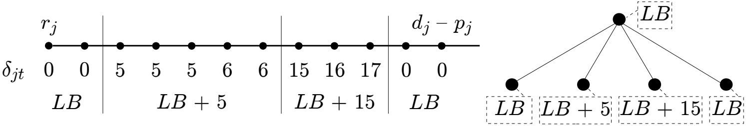
Strengthening lower bounds and determining branching alternatives

For each task j in turn, we recompute the lower bound by fixing task j in in each time point in the interval $[e_j, d_j - p_j]$.

Uniform partitioning:



Non-uniform partitioning:



Computational results

Average optimality gap for the linear objective function with $C_\ell = \lfloor \sum_{j \in N} b_{j\ell} p_j / D \rfloor$, and 3 resources.

	m5		m10		m20		avg	
	$CNST$	TS	$CNST$	TS	$CNST$	TS	$CNST$	TS
t10	20.72%	6.90%	8.98%	1.02%	9.04%	2.08%	**12.92%**	**3.33%**
t15	43.02%	13.47%	49.44%	6.78%	27.09%	1.82%	**39.85%**	**7.36%**
t20	55.28%	18.39%	49.80%	7.60%	39.94%	10.56%	**48.34%**	**12.18%**
avg	**39.68%**	**12.92%**	**36.07%**	**5.13%**	**25.36%**	**4.82%**	**33.70%**	**7.62%**

	m5		m10		m20		avg	
	BB	CPX	BB	CPX	BB	CPX	BB	CPX
t10	6.16%	3.10%	0.74%	0.24%	0.36%	0.37%	**2.42%**	**1.24%**
t15	12.94%	11.28%	5.08%	5.96%	0.41%	1.62%	**6.14%**	**6.29%**
t20	18.39%	17.15%	5.41%	7.48%	2.19%	10.39%	**8.66%**	**11.67%**
avg	**12.49%**	**10.51%**	**3.74%**	**4.56%**	**0.99%**	**4.13%**	**5.74%**	**6.40%**

Computational results (contd.)

Average optimality gap for the quadratic objective function with $C_\ell = 0$, and 3 resources.

	m5		m10		m20		avg	
	$CNST$	TS	$CNST$	TS	$CNST$	TS	$CNST$	TS
t10	6.83%	5.61%	5.09%	3.26%	4.63%	1.99%	**5.52%**	**3.62%**
t15	10.23%	8.13%	10.42%	7.50%	9.02%	5.27%	**9.89%**	**6.96%**
t20	11.53%	10.67%	10.89%	8.46%	9.43%	6.14%	**10.62%**	**8.43%**
avg	**9.53%**	**8.14%**	**8.80%**	**6.41%**	**7.69%**	**4.47%**	**8.67%**	**6.34%**

	m5		m10		m20		avg	
	BB	CPX	BB	CPX	BB	CPX	BB	CPX
t10	2.31%	1.51%	0.85%	–	0.24%	–	**1.13%**	–
t15	3.77%	–	1.20%	–	0.60%	–	**1.86%**	–
t20	4.31%	–	2.71%	–	0.37%	–	**2.46%**	–
avg	**3.46%**	–	**1.59%**	–	**0.40%**	–	**1.82%**	–

Application areas of resource leveling

1. Workforce optimization
   - Companies working with fixed and seasonal workers as well, may use workforce leveling to minimize hiring costs, and also to obtain a leveled use of workers
2. Energy optimization
   - If energy consumption is close to uniform throughout the machining operations, the energy usage may be leveled by the quadratic objective function.
   - If the goal is to minimize the total energy usage above a given limit, use the linear objective function.

Final remarks

1. Use of real-time data
    - Track jobs, stock of raw materials, other resources.
    - Some issues: accuracy, synchronization
2. Planning and scheduling over a rolling horizon
    - Instead of full reoptimization, change as little as possible, especially in the near future
    - Use real-time data when making a new schedule
3. Any new mathematical problems coming with the digital era?
    - Scheduling with material constraints: only a few theoretical results
    - Many types of resources in the same model, and multiple optimization criteria: very challenging to find provably good solutions