# Modeling of Material Flow Problems

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## Overview

1. Manufacturing Systems

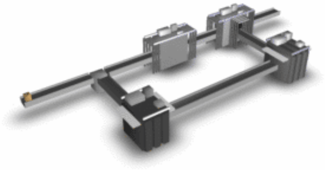
2. Workforce Determination

3. Material Flow Simulations

# Overview

# Introduction

- University of Mannheim:
  Dept. of Mathematics
- Strong focus on Business Mathematics
- **Industrial Partners:** BASF, Daimler AG
  $\Rightarrow$ Research interests:
  - Modeling and simulation of transportation networks (PDE and ODE)
  - Interaction of discrete and continuous optimization problems
  - Operations Research
- **Applications:** Manufacturing Systems, Traffic Flow, Pedestrian and Evacuation Dynamics, Power Grids



## Motivation

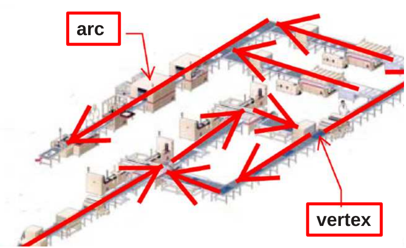
Siemens-Pressebild, ©Siemens AG

Pressebild DP, ©Deutsche Post AG

- Industrial manufacturing mostly consists of several production steps carried out by different processors
- Describe full dynamics of production processes (not only the steady state)

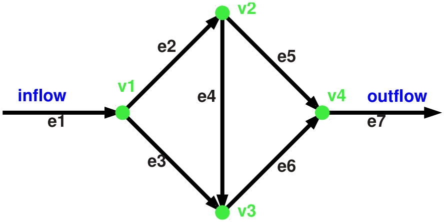
## Modeling of Manufacturing Systems

- **Assumptions:** Mass production, several production steps, consideration of inventory and processing
- **Goal:** Simulation of production dynamics, optimization studies with regard to costs and/or output



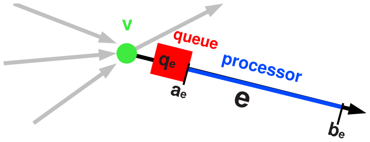
## Production Network Model

- **Basic setup:**
  - Production network $\Rightarrow$ directed, connected graph $G = (V, E)$
  - Machine / processor $\Rightarrow$ arc $e$
    parameters: length $L_e$, production speed $a_e$, max. capacity $\mu_e$
  - Distribution knot $\Rightarrow$ vertex $v$ with distribution rates $A^{v,e}$ into succeeding processors
  - Products $\Rightarrow$ continuous product density $\rho_e$
    (no discrete event simulation, DES)
    (dynamic model, no queueing theory)



## Production Network Model*

- **Queues:**
  - In front of any processor $e$: local storage queue $q_e$ (bounded or unbounded). Queue has no spatial extension.

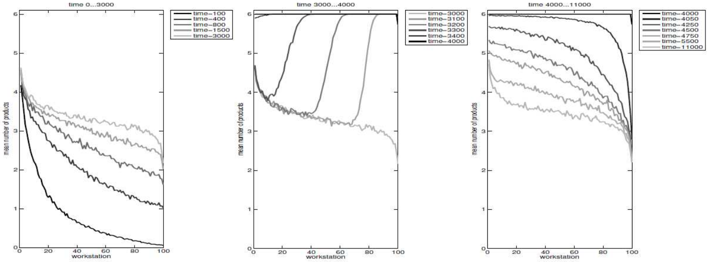



- **Model equations:**
  - Conservation law on processors (PDE):
$$\partial_t \rho_e(x,t) + \partial_x f_e(\rho_e(x,t)) = 0, \quad \forall x \in [0, L_e]$$
  - Balance equation for queue (ODE):
$$\partial_t q_e(t) = A^{v,e}(t) \sum_{\bar{e} \in \delta_v^{in}} f_{\bar{e}}^{in}(\rho_{\bar{e}}(x_{\bar{e}}^v, t)) - f_e^{out}(\rho_e(x_e^v, t))$$
  - Control-dependent distribution $A^{v,e}(t)$ of product flow

* with C. D'Apice, M. Herty, B. Piccoli (SIAM Math. Modeling and Computation, 2010)

## Clearing Functions*

... are the key idea of suitable production models. They can be obtained by fitting Discrete Event Simulation (DES) data.




©TU Eindhoven, $\chi$-DES Simulator by J.E. Rooda

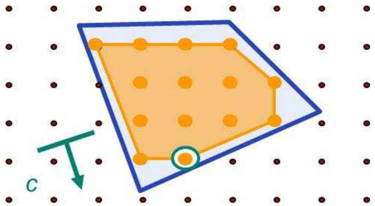
$$f(\rho, x) = H(\rho_{max} - \rho)\frac{\nu\rho}{1 + \rho + k\rho(1 - x)}$$

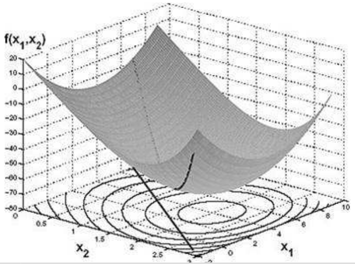
* with D. Armbruster, M. Herty (SIAM J. Appl. Math., 2011)

## Optimization Issues

- The presented models lead to a **PDE/ODE restricted optimization problems**, e.g. the optimal routing problem* or the buffer allocation problem**
- **Classical approach:** Solve the first order optimality system via steepest descent methods
- **Alternative approach:** Interpretation as a (time-dependent) mixed-integer problem (MIP)

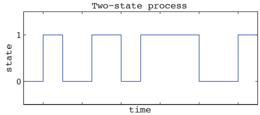
* with A. Fügenschuh et al. (SIAM J. Sci. Comput., 2008), ** with O. Kolb (submitted to European J. Oper. Res., 2014)



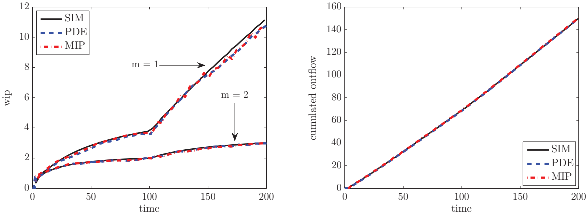
## Further Applications (Stochastics)

- **Continuous production networks with random breakdown*:** Modeling and simulation of exponentially distributed failures




* with S. Martin, T. Sickenberger (Netw. Heterog. Media, 2011)

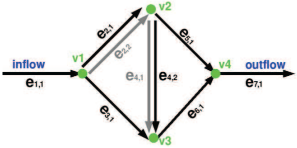
- **Approximations of time-dependent unreliable flow lines:** Derivation and validation of a continuous model from a stochastic MIP**




** with S. Kühn, J. Schwarz, R. Stolletz (submitted to European J. Oper. Res., 2013)

## Further Applications (Optimization/PDE)

- **Optimal Design of Capacitated Production Networks*:** Discrete decision problem with focus on minimal costs (setup, inventory, production). Initially, several configurations per arc are possible.




* with A. Dittel, U. Ziegler (Optim. Eng., 2011)

- **Production networks with finite buffers:** Limited buffer capacities $\rho_{max}$, extension to networks** and suitable numerics***

$$\partial_t \rho + \partial_x f(\rho, x) = 0, \quad f(\rho, x) = \tilde{f}(\rho, x) H(\rho_{\max} - \rho),$$

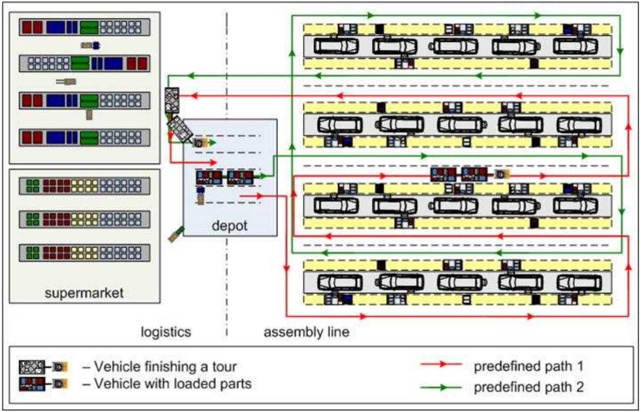
where $\tilde{f}(\rho, x)$ is smooth and concave

** with A. Klar, P. Schindler (SIAM J. Appl. Math., 2013),

*** with P. Schindler (submitted to Discrete Contin. Dyn. Syst. Ser. B, 2014)

# Overview

# Motivation*

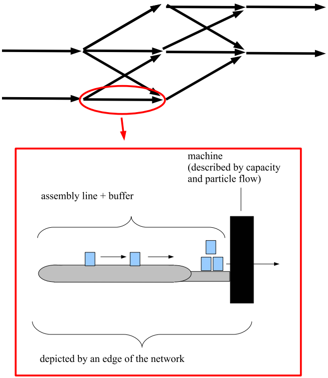



- **Goal:** Workforce scheduling during the production process

* joint work with M. Herty (RWTH Aachen), U. Ziegler (University of Mannheim), Marcus Ziegler (Daimler AG)

## Workforce Determination in Production Networks*

- Given a directed graph $G = (V, E)$. Due to machine failures capacity drops can occur.
- **Goal:** Allocate service operators to still maximize the production output.
- Workforce planning highly relevant for maintenance and cost optimization
  $\Rightarrow$ Model should include:
  - service operation determination
  - machine-individual default parameters
  - evolution of capacity,
  - reliable optimization procedures




* with M. Herty, C. Ringhofer, U. Ziegler (Optimization, 2012)

## Model Equations

Each production unit is described by a set of functions:

- $w_e(t)$: number of workers at production unit
- $q_e(t)$: buffer level in front of machine $e$,
  coupled ODE depends on inflow and outflow of buffer
- $c_e(t)$: current processing capacity of machine $e$,
  ODE depends on properties of production unit and number of workers
- $f_e(t)$: flow through machine $e$
  $\Rightarrow$ coupled ODE depends on capacity and buffer level

$$\partial_t q_e = Bf + f_{in} - f_e, \quad f_e = \min\{c_e, q_e/\tau_e\}$$

$$\partial_t c_e = -\min\{\frac{c_e}{\epsilon}, \alpha_e\} + \min\{\frac{\mu_e - c_e}{\epsilon}, w_e\}$$

## Service Operator Assignment

- Given a service operator schedule $w_e(t)$ and an inflow function $f_{in}(t)$ into the system.
- All service operators are distributed among the machines:

$$\sum_e w_e(t) = W, \quad w_e(t) \geq 0$$

- Highly fluctuating worker schedules are not applicable $\rightarrow$ $w_e(t)$ should be either constant or piecewise constant. (i.e. service operators can shift within the time horizon.)
- Restrict to integer workers:

$$w_e \in \mathbb{N}_0$$

## Mixed Integer Approach

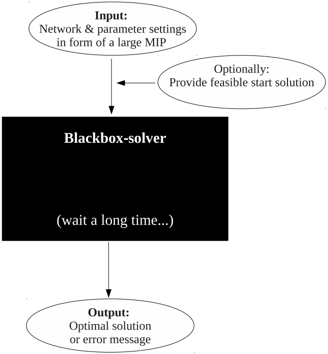
- The objective is to maximize the outflow of the network:

$$\max \sum_e \int f_e(t)dt$$

- Full discretization and linearization of the problem
- $\rightarrow$ Transformation into a linear MIP
  - Solvers with integrated Branch and Bound Algorithms applicable (e.g. CPLEX)
  - **Benefit:** easy expansion by further constraints, e.g. buffer limits, service operator shifts,...
  - **Drawback:** worst case computational time is exponentially to the problem size

# Challenges and improvements in solving complex linear MIPs*

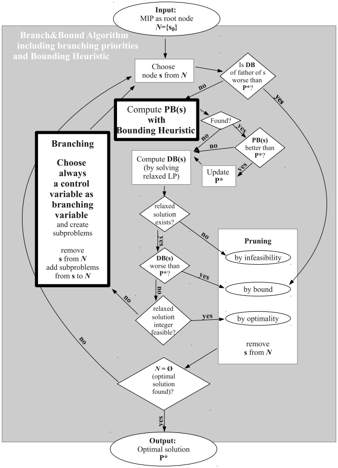
- Complexity of the MIP depends on number of gridpoints.
- Extremely long computation times.
- Fine timegrids $\Rightarrow$ accumulation of rounding errors $\Rightarrow$ CPLEX unable to find feasible solutions.
- Idea: Exploiting knowledge of dynamics to easily compute feasible solutions
- Utilization of Start-Heuristics
- Further strategies in using heuristics during branch and bound process.



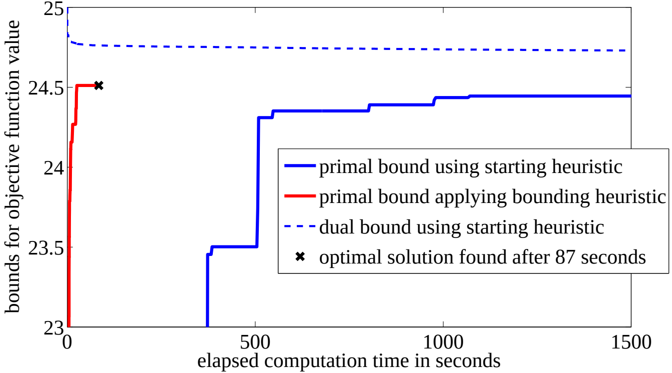

* with U. Ziegler (in preparation, 2014)

# Acceleration of Branch and Bound Algorithm

- Branching Heuristic:
  Choose only worker parameters $w_e(t)$ for branching.
- Bounding Heuristic: Find "good" feasible worker distribution:
  - Consider the index set whose control variables are not fixed yet
  - Sort arcs according to buffer level at last time step of shift $q_e^{\hat{t}}$
  - Add one by one all remaining workers in this order to the arcs



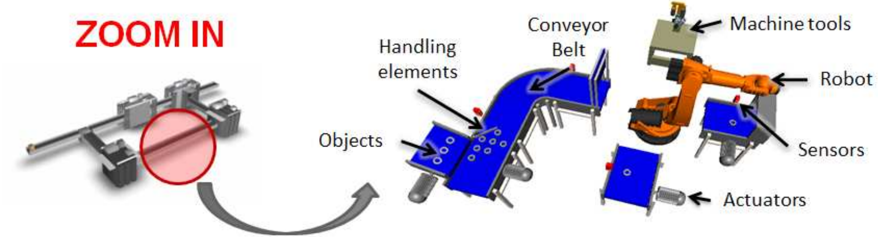
# Evolution of Primal and Dual Bounds




- Small network instance $\approx$ 5000 variables: Optimization time without bounding heuristic needs 51 minutes

## Overview

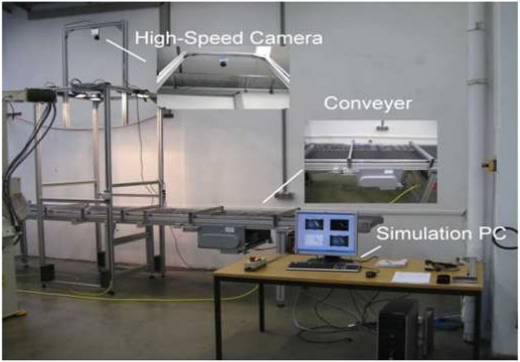
## Motivation*




- **Goal:** Detailed 2d simulation of parts on conveyor belts

* joint work with V. Schleper (University of Stuttgart), A. Verl (ISW Stuttgart)

# Experimental Setup




## Microscopic Model

- **Newton type dynamics**

$$\frac{d\mathbf{x}_i}{dt} = \mathbf{v}_i, \quad m\frac{d\mathbf{v}_i}{dt} = \sum_{i \neq j} \mathbf{F}(\mathbf{x}_i - \mathbf{x}_j, \mathbf{v}_i - \mathbf{v}_j) + \mathbf{G}(\mathbf{v}_i)$$

- Bottom friction

$$\mathbf{G}(\mathbf{v}) = -\mu_b m g \frac{\mathbf{v} - \mathbf{v_T}}{\|\mathbf{v} - \mathbf{v_T}\|}$$

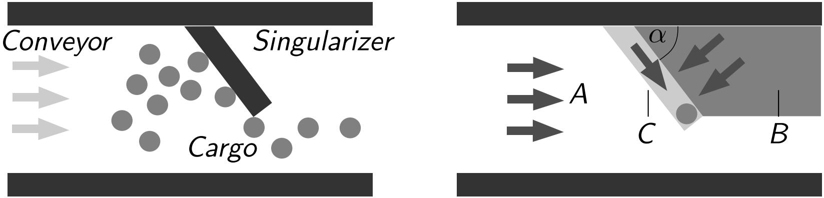
- Springer-damper system

$$\mathbf{F}(\mathbf{x}, \mathbf{v}) = H(2R - \|\mathbf{x}\|) \left(\mathbf{f^n}(\mathbf{x}, \mathbf{v}) + \mathbf{f^t}(\mathbf{x}, \mathbf{v})\right)$$

**Goal:** Try to find a macroscopic model with “same” physical properties

## Macroscopic Model*

**Goal:** Numerical study and validation against real data



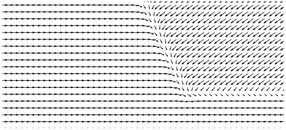

- **Conservation of Mass**

$$\partial_t \rho + \nabla \cdot (\rho(\mathbf{v}^{dyn}(\rho) + \mathbf{v}^{stat}(\mathbf{x}))) = 0$$
$$\rho(\mathbf{x}, 0) = \rho_0(\mathbf{x}), \quad \mathbf{x} \in \mathbb{R}^2$$

- Static velocity field $\mathbf{v}^{stat}$

* with V. Schleper, A. Verl et al. (to appear in Appl. Math. Modelling, 2014)

# Macroscopic Model



- Dynamic velocity field $\mathbf{v}^{dyn}$

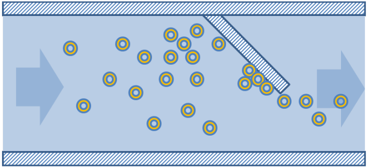
$$\mathbf{v}^{dyn}(\rho) = H(\rho - \rho_{max}) \cdot \mathbf{I}(\rho), \quad \mathbf{I}(\rho) = -\epsilon \frac{\nabla(\eta * \rho)}{\sqrt{1 + \|\nabla(\eta * \rho)\|_2^2}},$$

where $\eta$ is a mollifier and $\epsilon > 0$ a constant.

- Possible to deribve a multi-scale model hierarchy*

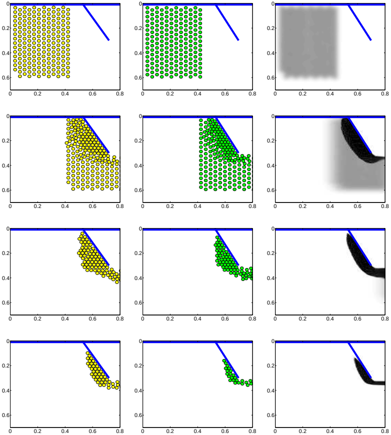
* with A. Klar, S. Tiwari (eingereicht bei J. Engrg. Math., 2013)

# Experiment



- **Experiment:**
  - 192 objects
  - Angle of the singularizer $\alpha = 60^\circ$
  - Speed of conveyor belt $v_T = 0.395 \frac{m}{s}$

## Simulation Results




- Real data (left), micro model (middle) and macro model (right) at times $t = 0, 1, 2, 3$.

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  no interaction between scales $\Rightarrow$ hybrid models?
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- **Stochastics:** How does stochastic effects influence the model?

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- **Stochastics:** How does stochastic effects influence the model?
  unclear ....

# Thank you for your kind attention!

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