L. Ghezzi, A. Sciacca, F. Rigamonti / ABB Italy – Mathematics for the Digital Factory, Berlin May 2014

Applications of Computational Mathematics to Industrial Products and Processes
The Case of ABB

Contents

- ABB company overview
- Applications to products
- Applications to processes
- Future research trends

ABB Company Overview

Power and productivity
for a better world™

A global leader in power and automation technologies
Leading market positions in main businesses

~150,000
employees

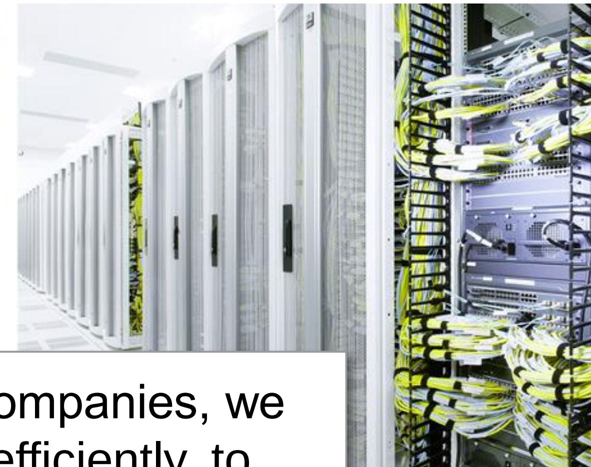
$42 billion
In revenue
(2013)

Present
in
+100
countries

Formed
in
1988
merger of Swiss (BBC, 1891)
and Swedish (ASEA, 1883)
engineering companies

Power and productivity for a better world
ABB's vision

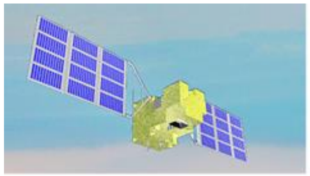




As one of the world's leading engineering companies, we help our customers to use electrical power efficiently, to increase industrial productivity and to lower environmental impact in a sustainable way.



Power and automation are all around us
You will find ABB technology…




orbiting the earth and working beneath it,

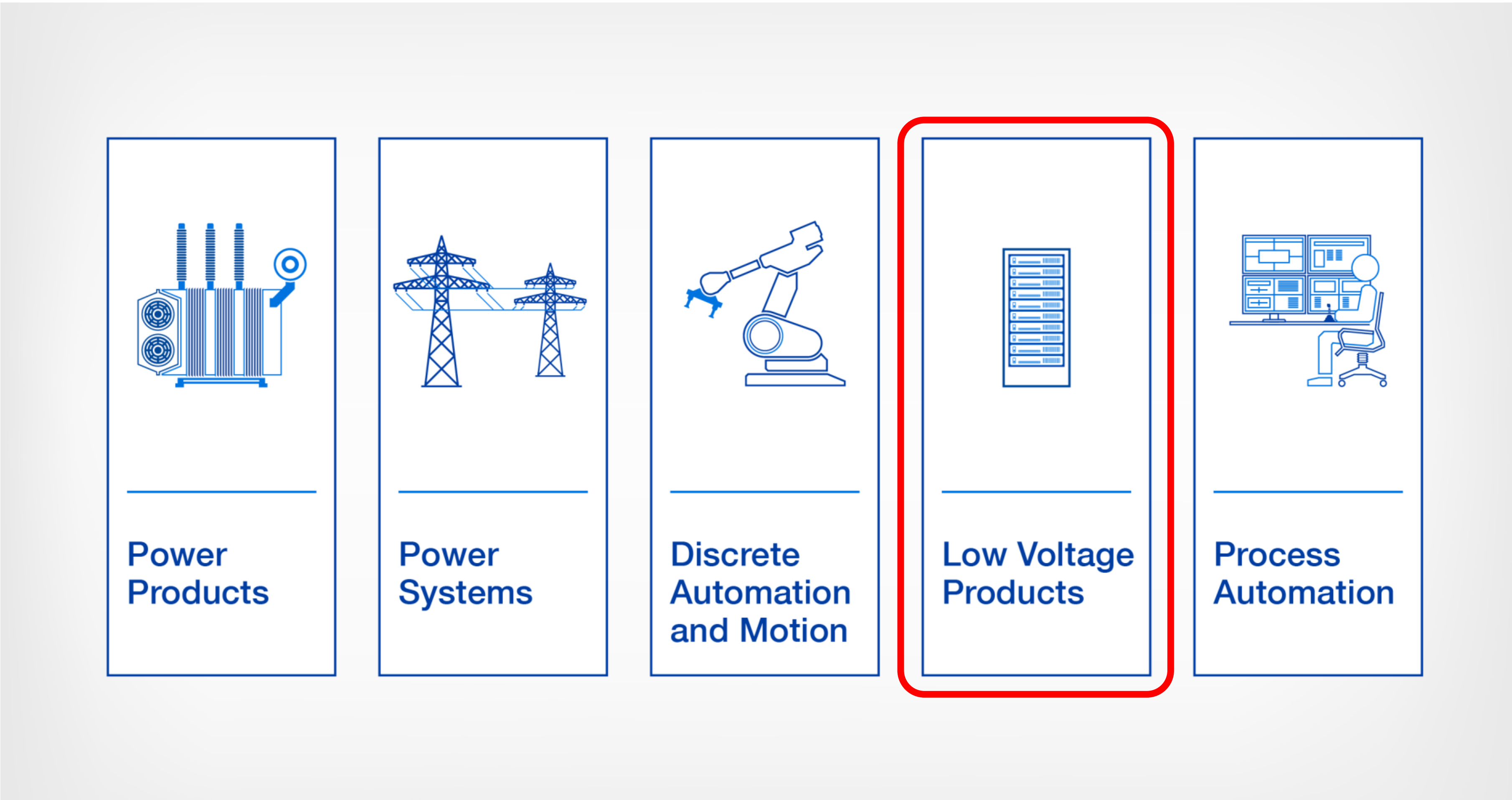

crossing oceans and on the sea bed,


in the fields that grow our crops and packing the food we eat,


on the trains we ride and in the facilities that process our water,


in the plants that generate our power and in our homes, offices and factories

How ABB is organized
Five global divisions



Low Voltage Products
Business Units

LV Systems

- MNS Conventional Switchgear
- MNS Intelligent Switchgear
- MNS Integrated Switchgear

Breakers and Switches

- Circuit Breakers
- Switches
- Fusegear and Cable Distribution Cabinets

Enclosures and DIN-Rail Products

- Modular DIN-Rail Products
- Intelligent Building Control KNX
- Enclosures and Cable Systems

Control Products

- Control and Protection
- Electronic Products and Relays
- Connection

Wiring Accessories

- Wiring Accessories
- Industrial Plugs and Sockets
- Door Entry Systems

Thomas & Betts

- Wire and cable management
- Cable protection systems
- Critical process protection
- People protection

Low Voltage Products
Channels and Markets served

Distributors

Panel Builders

OEMs

System Integrators and Contractors

End-users and Utilities

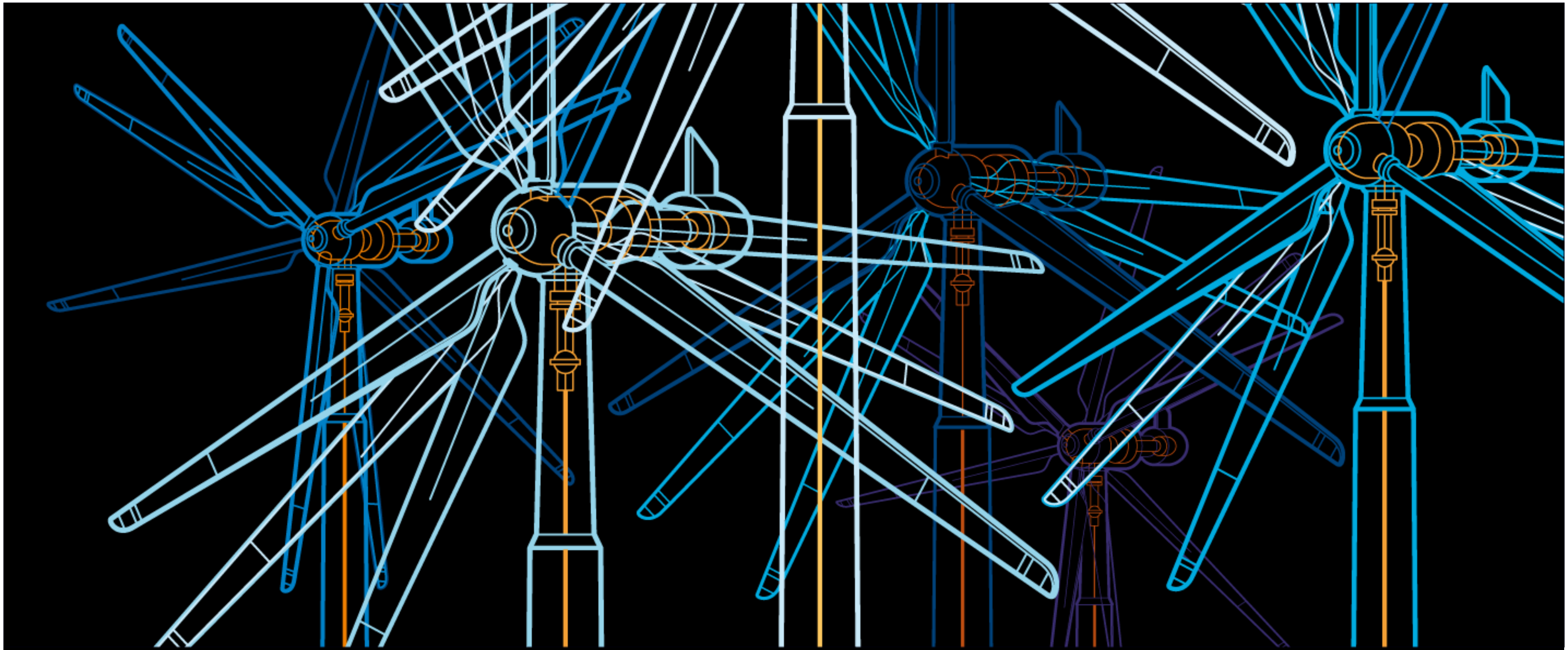
Industry

Buildings and infrastructures

Renewable energy

E-mobility

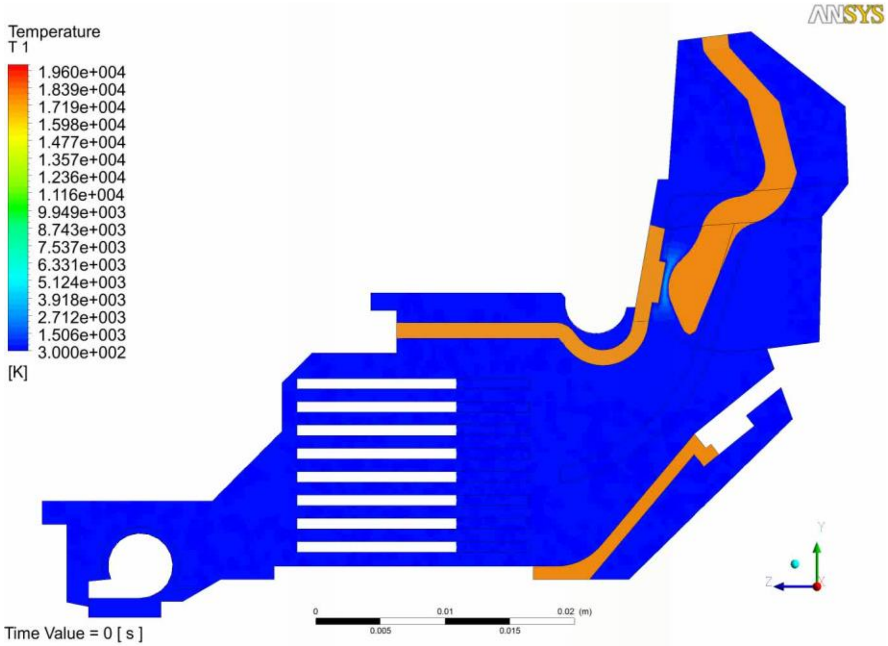
Building Automation and Energy Efficiency



Applications to Products

Computational Magnetohydrodynamics (MHD)
Numerical simulation of electric arcs

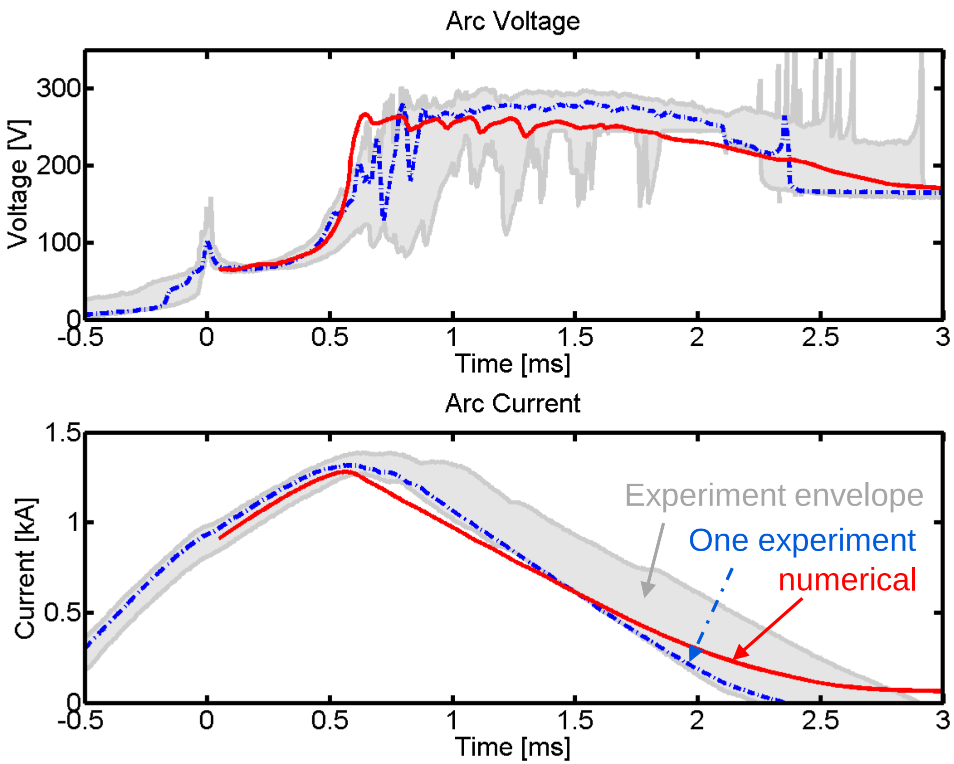
Short circuits in
low voltage circuit breakers



- Navier-Stokes PDE's
- Maxwell PDE's
- Radiation PDE's
- Network DAE's
- Nonconstant media
- Plasma-solid interface

Computational Magnetohydrodynamics (MHD)
Numerical simulation of electric arcs

Short circuit prediction vs. measured



- Navier-Stokes PDE's
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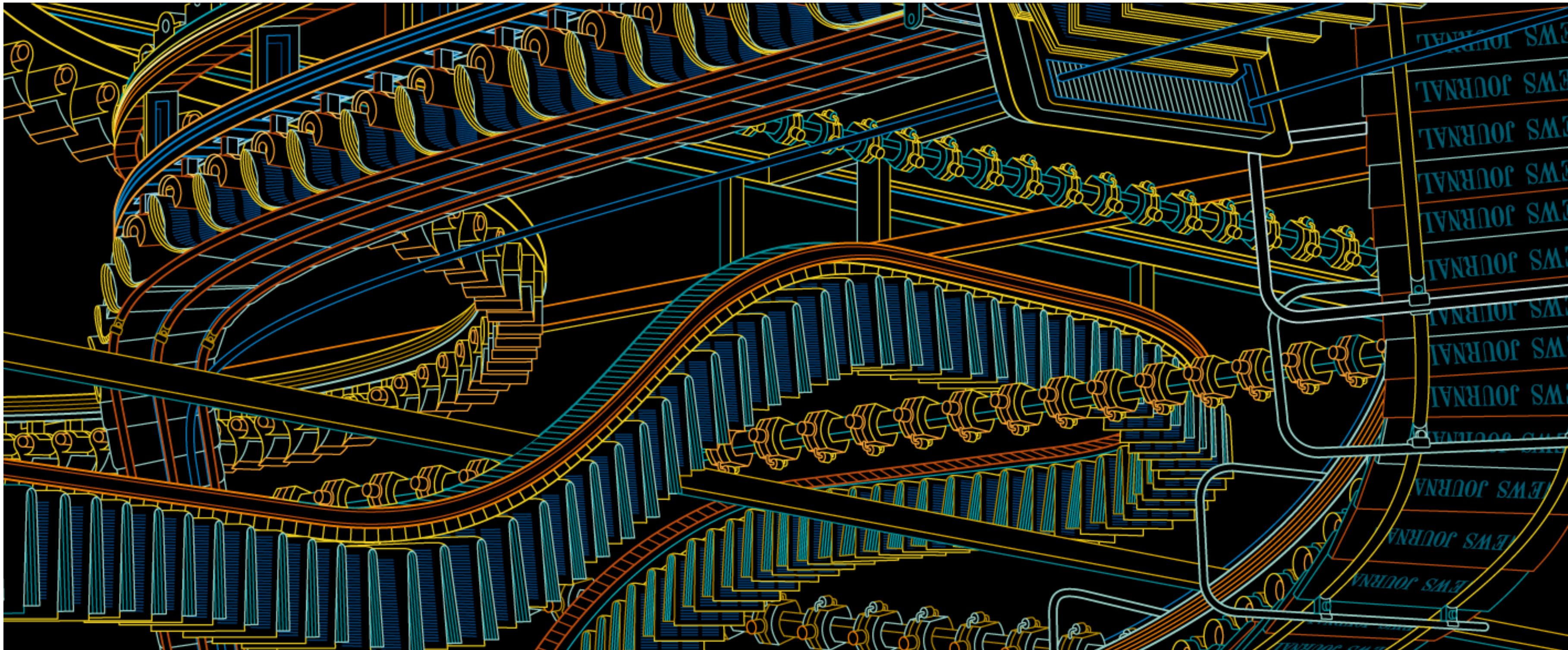
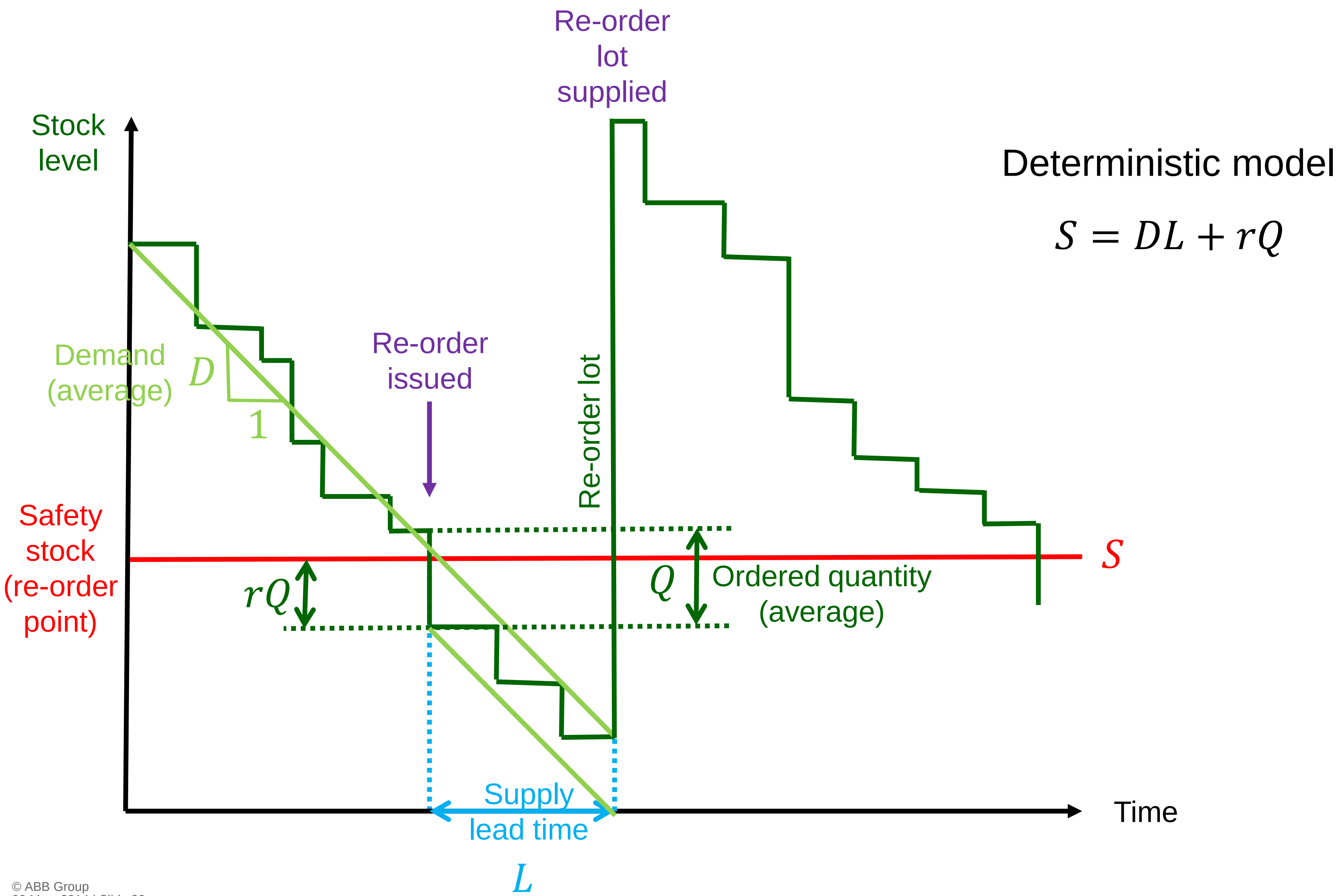
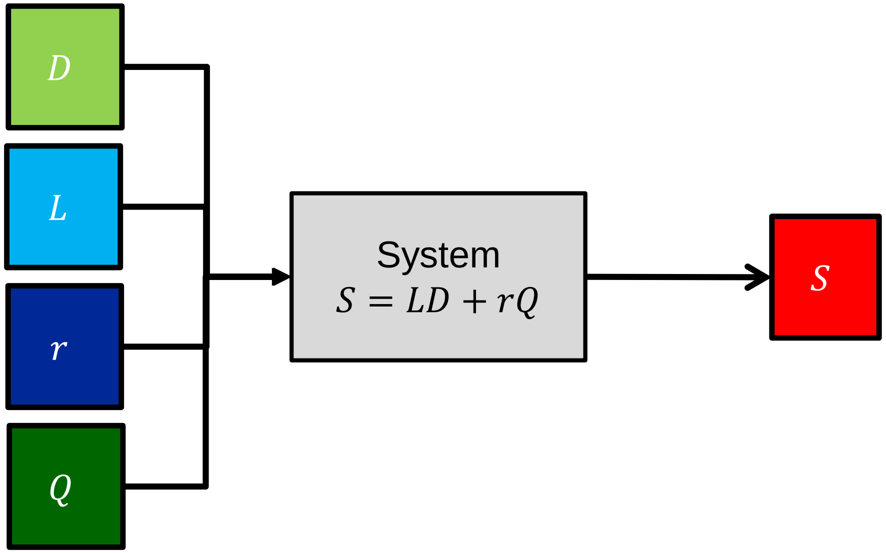


Applications to Processes

Stock sizing
Overview

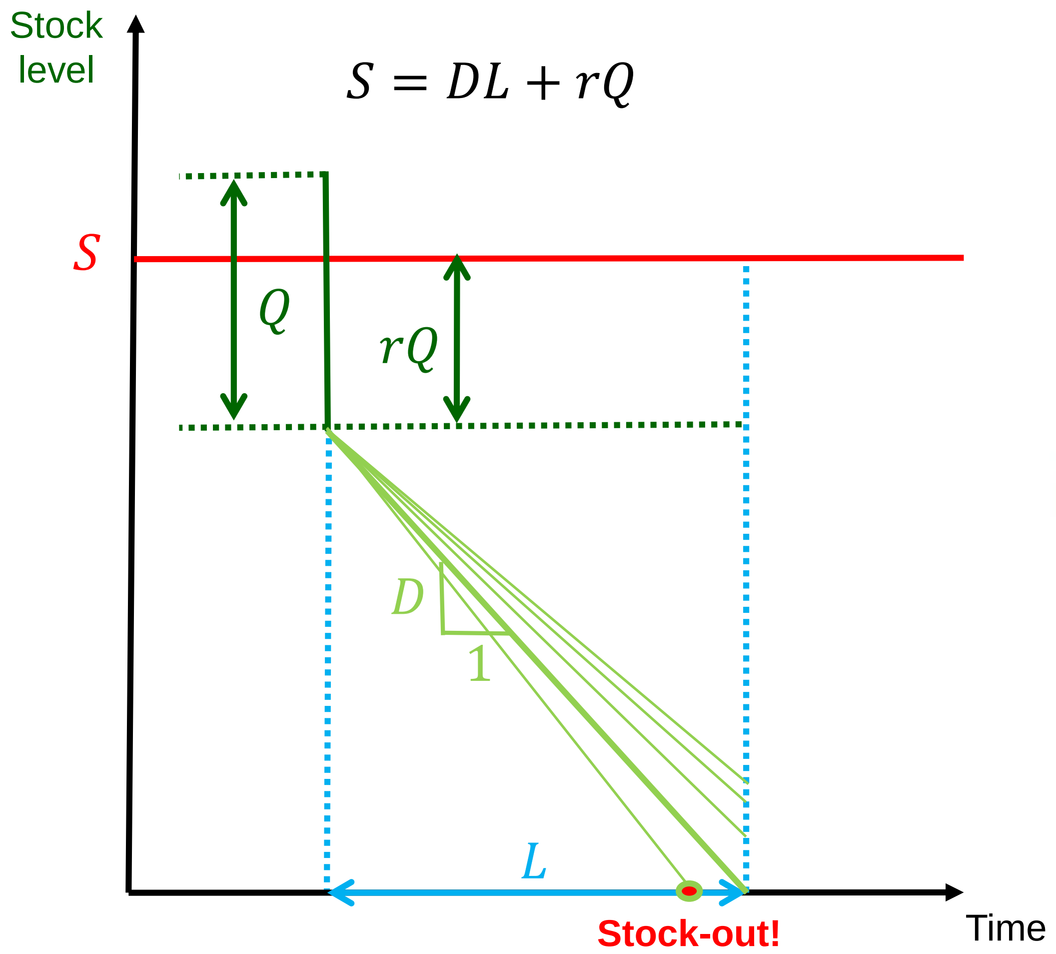


Deterministic approach

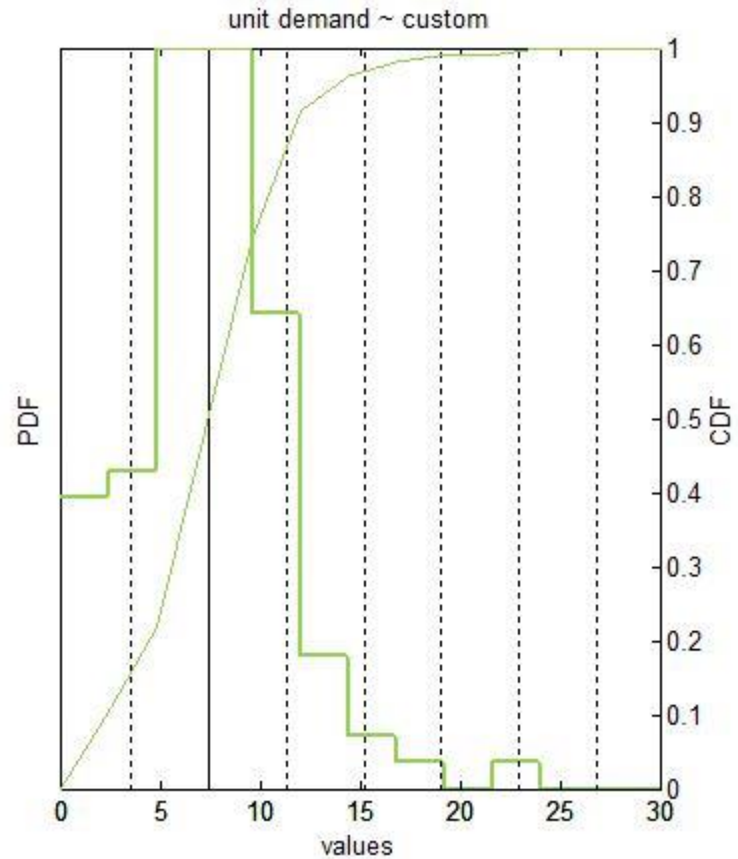


- Everything sharp
- System may be mathematical, or black box, or else

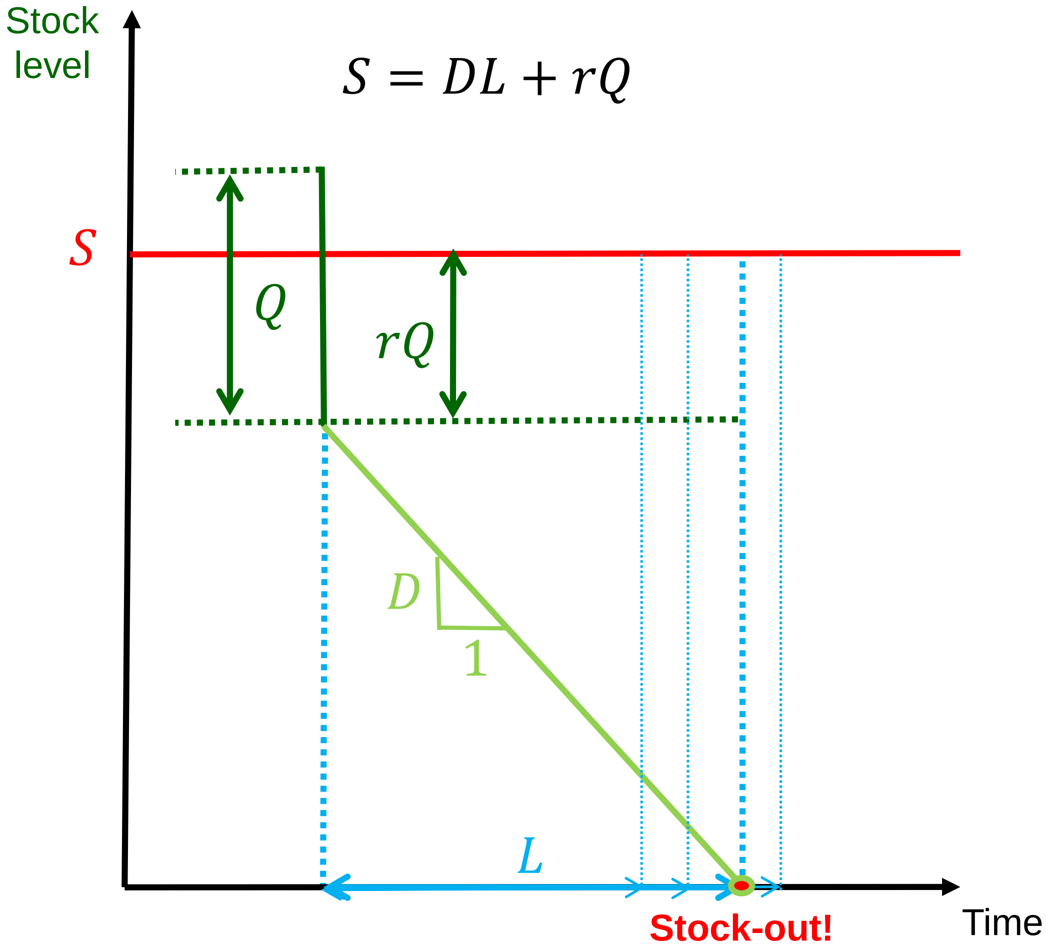
Stock sizing
Stochastic problem



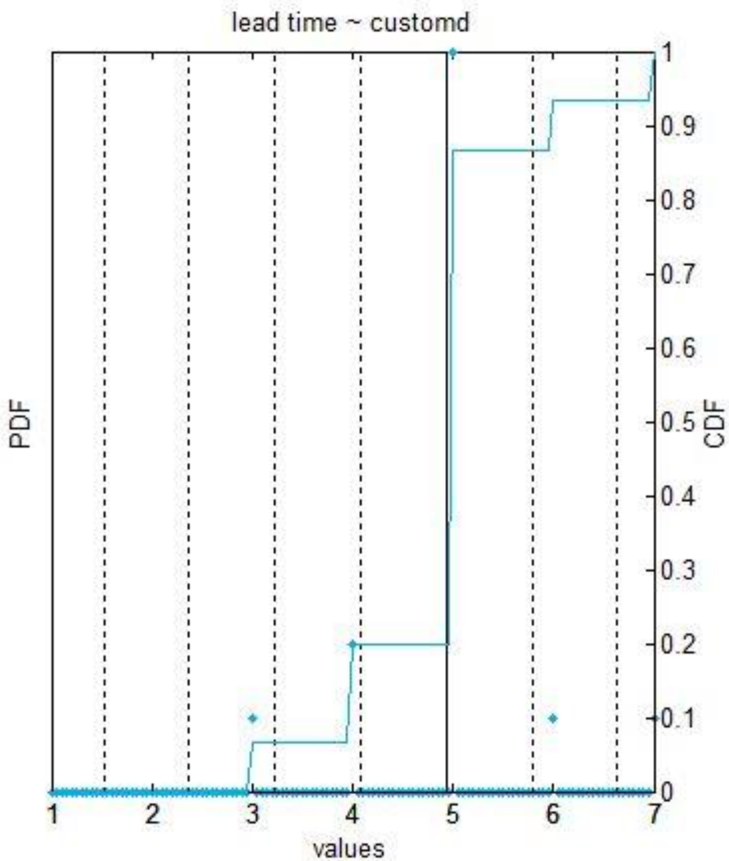
D as a Stochastic variable



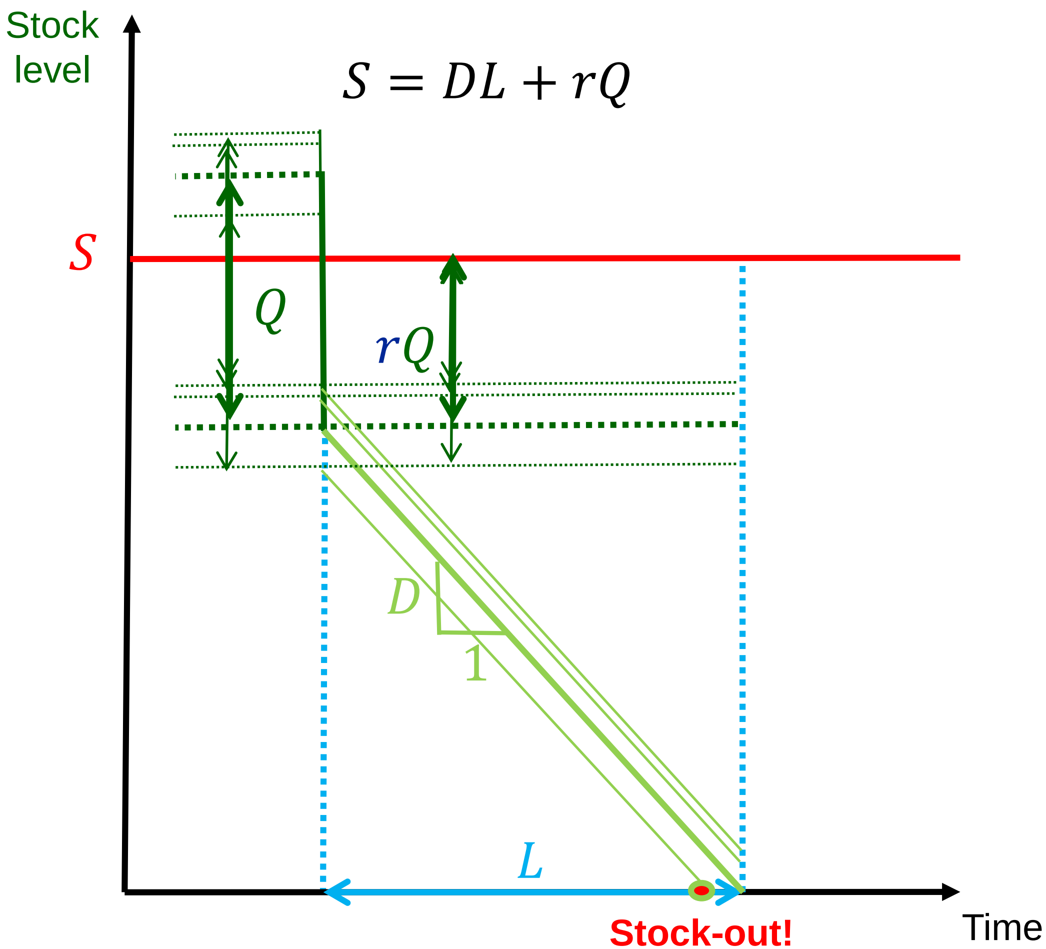
Stock sizing
Stochastic problem



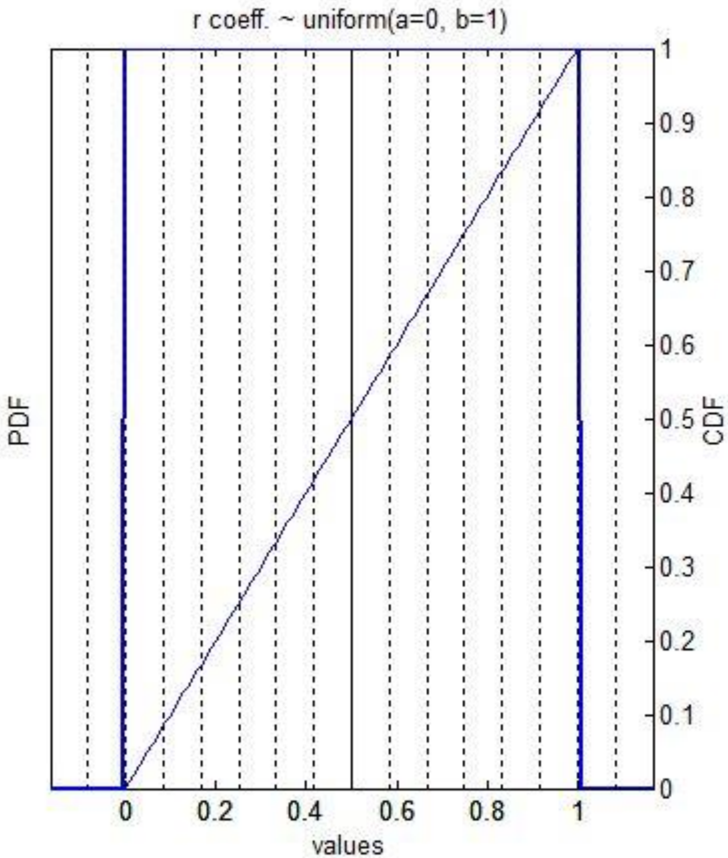
L as a Stochastic variable



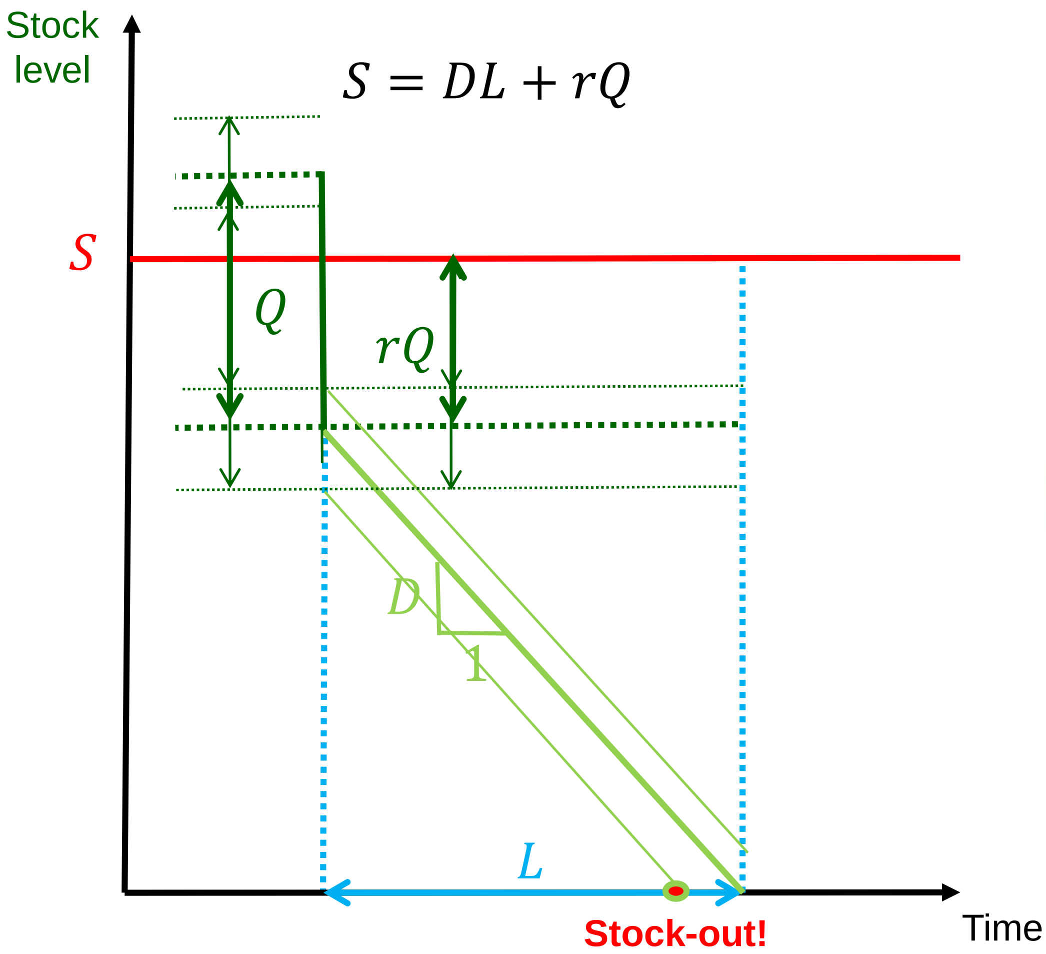
Stock sizing
Stochastic problem



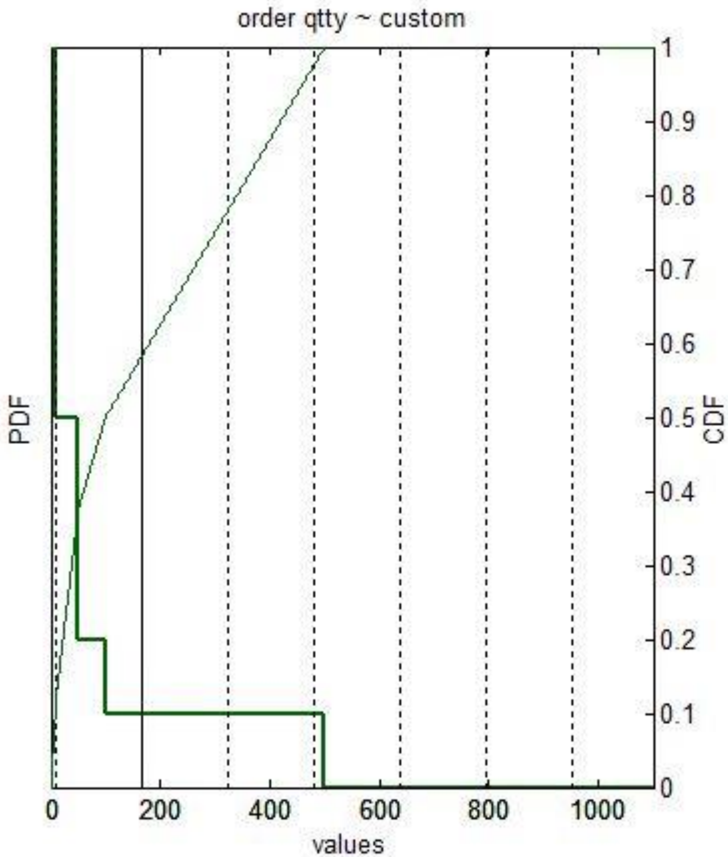
r as a Stochastic variable



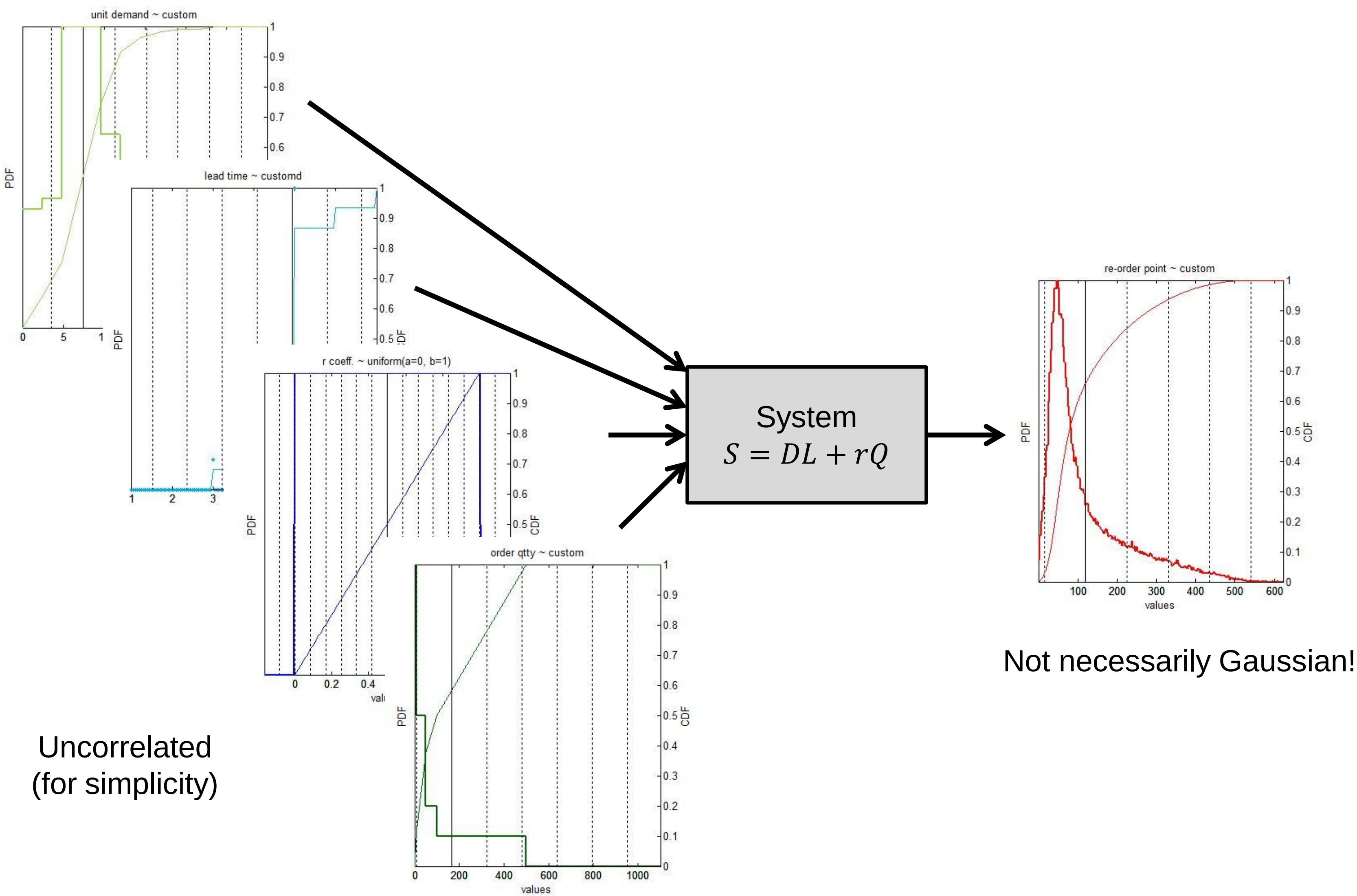
Stock sizing
Stochastic problem



Q as a Stochastic variable



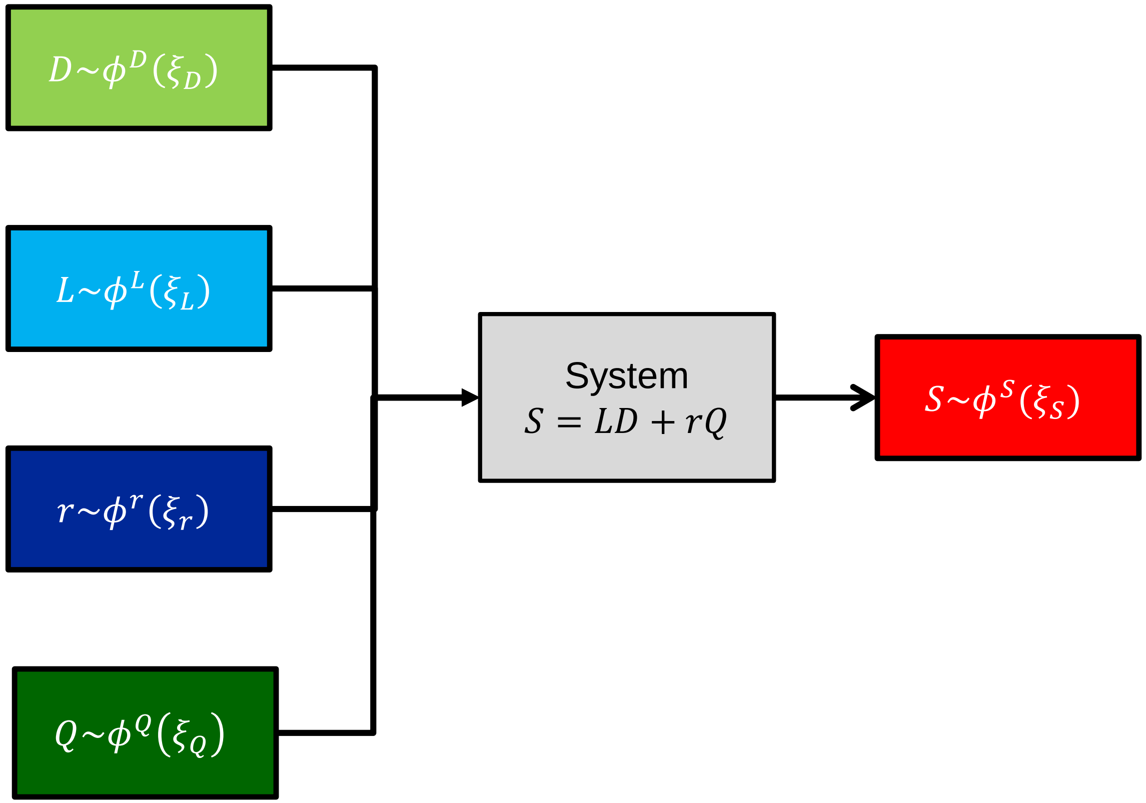
Stochastic approach



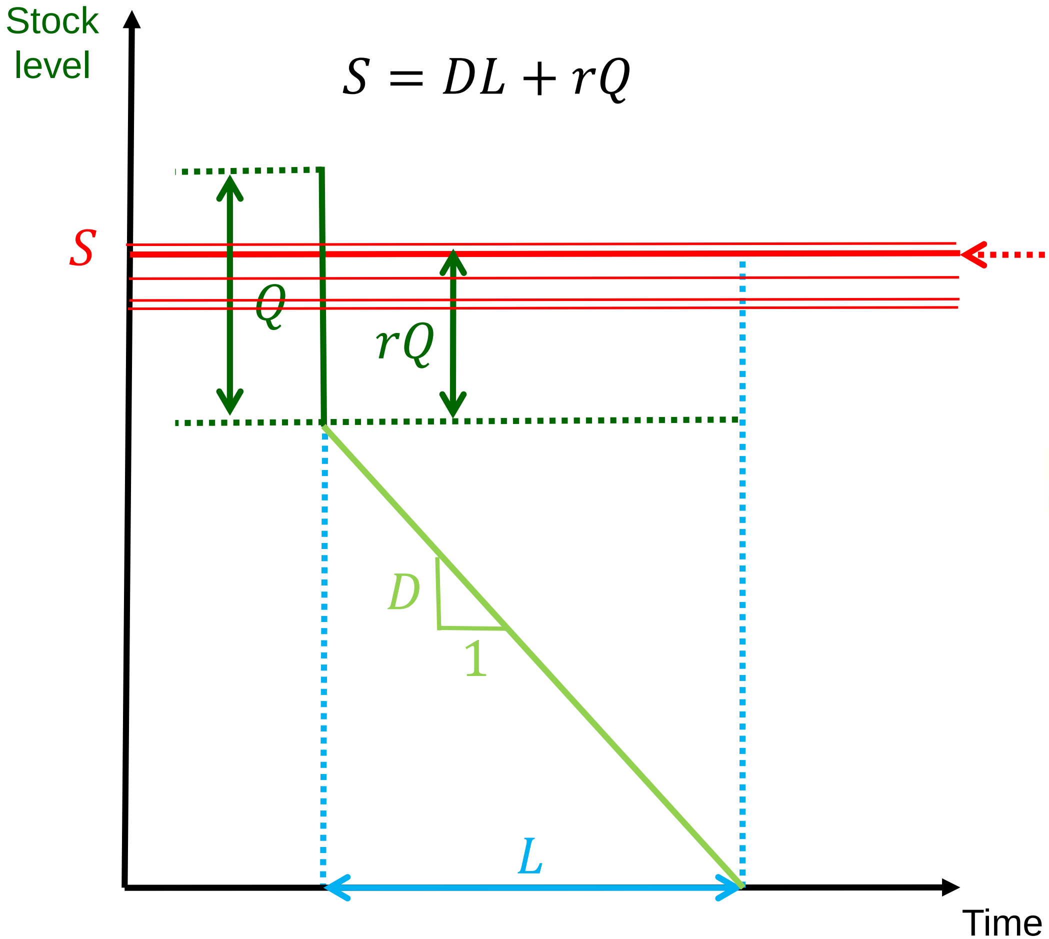
Not necessarily Gaussian!

Uncorrelated
(for simplicity)

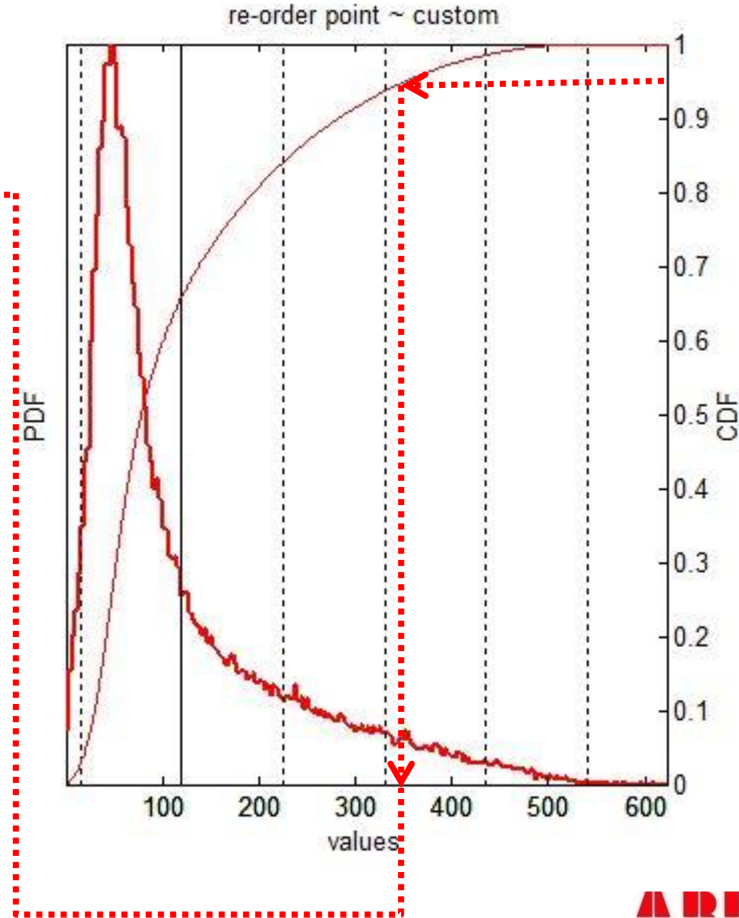
Stochastic approach



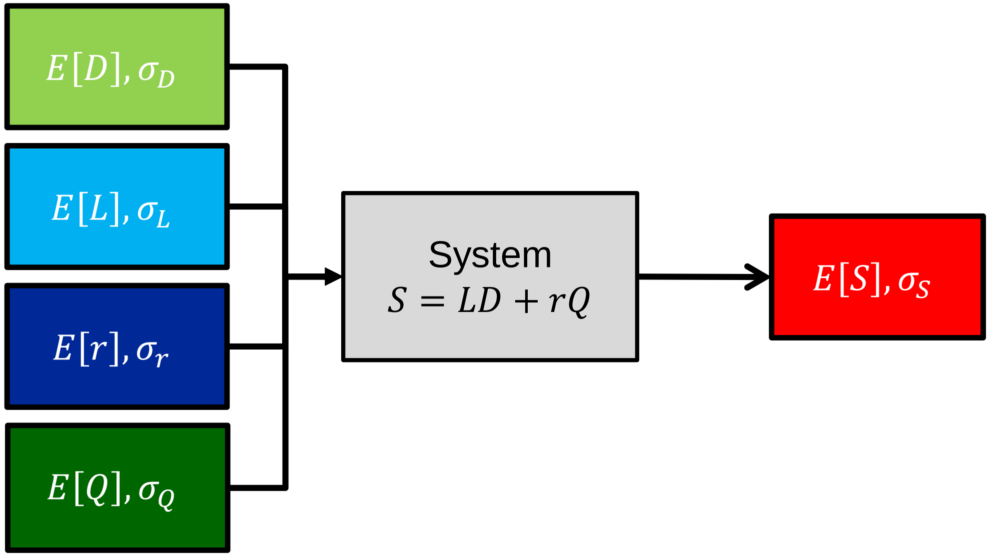
Stock sizing
Stochastic problem



S as a Stochastic variable

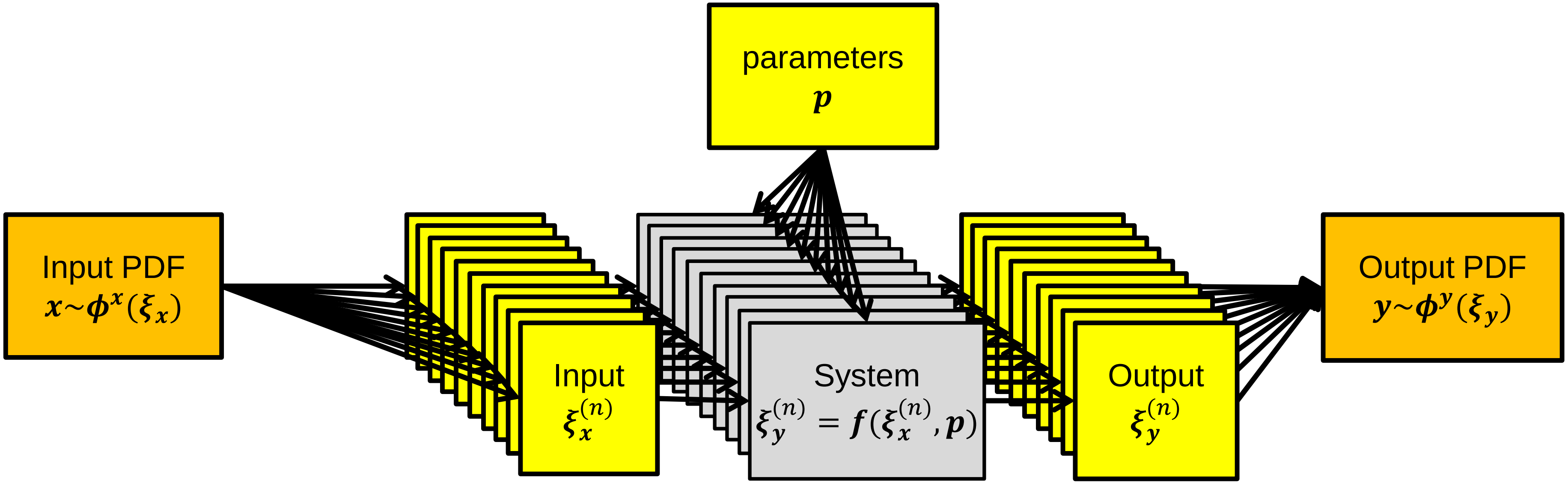


Stochastic approach
Traditional



- System linearization $S := E[L]E[D] + E[r]E[Q]$
- Variance propagation $\sigma_S = \sqrt{L^2\sigma_D^2 + D^2\sigma_L^2 + Q^2\sigma_r^2 + r^2\,\sigma_Q^2}$
- Output PDF assumed Gaussian, usually unproperly!

Stochastic approach
Monte Carlo



- Simple
- Non-intrusive
- Parallel
- Large number of stochastic variables allowed
- Slow, very slow(!): 10^6 (or even more) runs can be required

Orthogonal Polynomial Sequences (OPS)

- Probability density distribution (→ metric)

$$x \sim \phi^x(\xi)$$

- Inner product in $L^2(\mathbb{R})$

$$\big(u(\xi), v(\xi)\big)_x := E[uv] = \int_{\mathbb{R}} u(\xi) \cdot v(\xi) \cdot \phi_k^x(\xi) d\xi$$

- Gram-Schmidt process applied to $1, \xi, \xi^2, \ldots, \xi^k, \ldots$

→ polynomials $\{\psi_k^x(\xi)\}_{k=0}^{+\infty}$

Orthogonal Polynomial Sequences (OPS)

- Polynomials are independent (degree argument)
- Polynomals are orthogonal (or orthonormal, if desired)

$$\left(\psi_j^x, \psi_k^x\right)_x \coloneqq E\left[\psi_j^x \psi_k^x\right] = \int_{\mathbb{R}} \psi_j^x(\xi) \cdot \psi_k^x(\xi) \cdot \phi_k^x(\xi) d\xi = \|\psi_j^x\|_x \|\psi_k^x\|_x \delta_{jk}$$

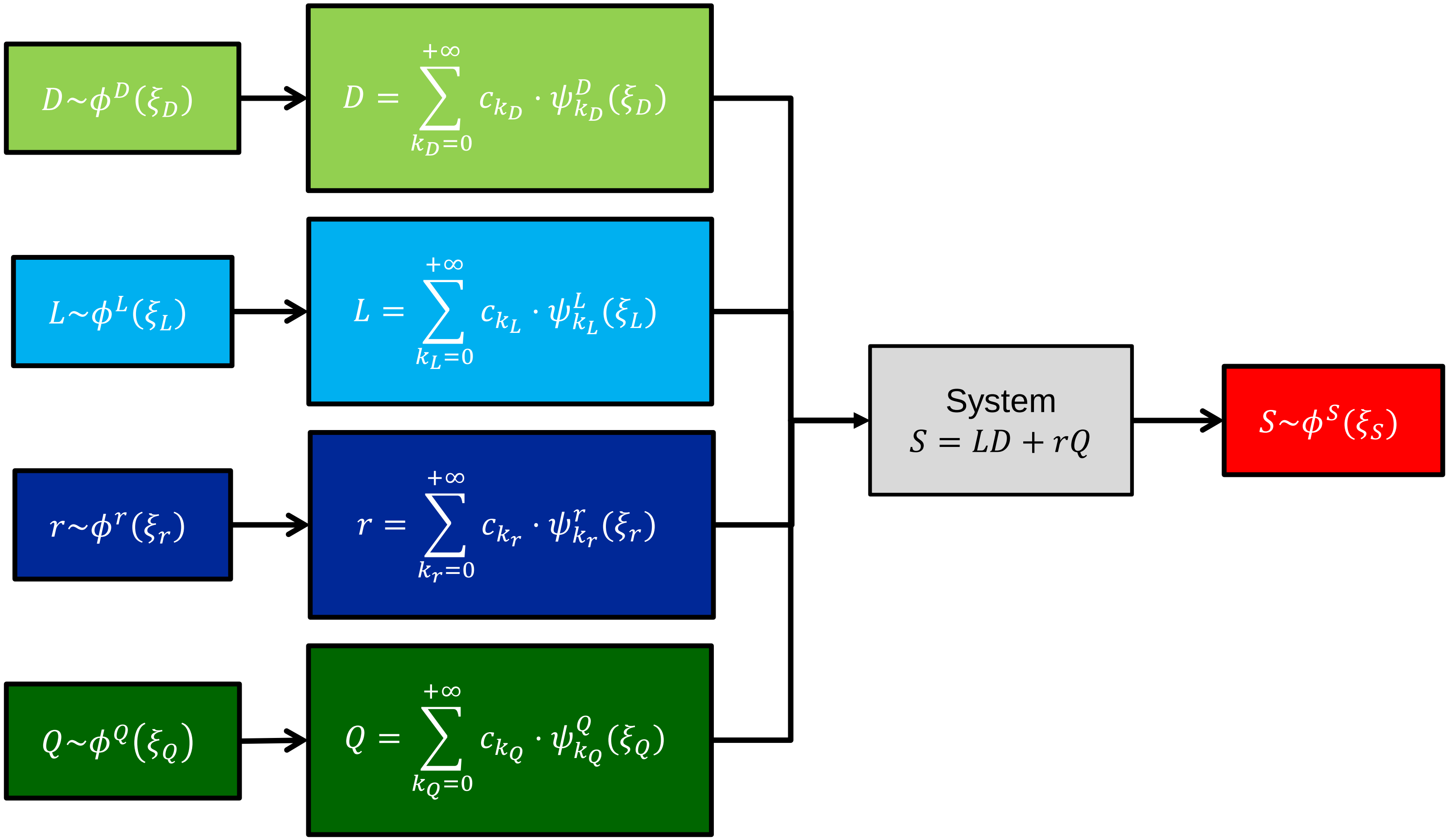
- Polynomials form a basis of $L^2(\mathbb{R})$

$$x(\xi) = \sum_{k=1}^{+\infty} c_k \cdot \psi_k^x(\xi)$$

- Fourier series expansion

$$c_k = \frac{\left(x(\xi), \psi_k^x(\xi)\right)_x}{\left(\psi_k^x(\xi), \psi_k^x(\xi)\right)_x} = \frac{E\left[x \psi_k^x\right]}{\|\psi_k^x\|_x^2}$$

Stochastic approach
Polynomial Chaos Expansion (PCE)



Stochastic approach
Polynomial Chaos Expansion (PCE)

to be truncated to P

$\boldsymbol{\xi} = (\xi_D, \xi_L, \xi_r, \xi_Q)$

Surrogate model

$$S = S(\boldsymbol{\xi}) = \sum_{|\boldsymbol{k}|=0}^{+\infty} c_{\boldsymbol{k}} \cdot \psi_{\boldsymbol{k}}^{S}(\boldsymbol{\xi})$$

(analytical, by orthogonality)

$$E[S] = c_{\mathbf{0}} \left\| \psi_{\mathbf{0}}^{S} \right\|_{\boldsymbol{x}}^{2}$$

$$\sigma_S = \sum_{|\boldsymbol{k}|=0}^{P} c_{\boldsymbol{k}}^{2} \left\| \psi_{\boldsymbol{k}}^{S} \right\|_{\boldsymbol{x}}^{2}$$

(comput. by, e.g., fast MC)

$$S \sim \phi^{S}(\xi_S)$$

$\boldsymbol{k} = (k_D, k_L, k_r, k_Q)$

multi-index

$|\boldsymbol{k}| = k_D + k_L + k_r + k_Q$

By tensor product

$$\psi_{\boldsymbol{k}}^{S}(\boldsymbol{\xi}) \coloneqq \psi_{\boldsymbol{k}}^{S}(\xi_D, \xi_L, \xi_r, \xi_Q) = \psi_{k_D}^{D}(\xi_D) \cdot \psi_{k_L}^{L}(\xi_L) \cdot \psi_{k_r}^{r}(\xi_r) \cdot \psi_{k_Q}^{Q}(\xi_Q)$$

Orthogonality inheritance

$$E\left[\psi_{\boldsymbol{j}}^{S} \psi_{\boldsymbol{k}}^{S}\right] = \left\| \psi_{\boldsymbol{j}}^{S} \right\|_{\boldsymbol{x}} \left\| \psi_{\boldsymbol{k}}^{S} \right\|_{\boldsymbol{x}} \delta_{\boldsymbol{jk}} = \begin{cases} 1 & (\boldsymbol{j} = \boldsymbol{k}) \\ 0 & (\boldsymbol{j} \neq \boldsymbol{k}) \end{cases}$$

Stochastic approach
Polynomial Chaos Expansion (PCE)
Non-intrusive approach (collocation)

$$S = S(\boldsymbol{\xi}) = \sum_{|\boldsymbol{k}|=0}^{P} c_{\boldsymbol{k}} \cdot \psi_{\boldsymbol{k}}^{S}(\boldsymbol{\xi})$$

System
$S = LD + rQ$

$$c_k = \frac{\left(S(\boldsymbol{\xi}), \psi_{\boldsymbol{k}}^{S}(\boldsymbol{\xi})\right)_{\boldsymbol{x}}}{\left(\psi_{\boldsymbol{k}}^{S}(\boldsymbol{\xi}), \psi_{\boldsymbol{k}}^{S}(\boldsymbol{\xi})\right)_{\boldsymbol{x}}} = \frac{E\left[S(\boldsymbol{\xi})\psi_{\boldsymbol{k}}^{S}(\boldsymbol{\xi})\right]}{\left\|\psi_{\boldsymbol{k}}^{S}\right\|_{\boldsymbol{x}}^{2}} = \frac{\sum_{j=1}^{N} w_j S\left(\boldsymbol{\xi}_j\right) \psi_{\boldsymbol{k}}^{S}(\boldsymbol{\xi}_j)}{\left\|\psi_{\boldsymbol{k}}^{S}\right\|_{\boldsymbol{x}}^{2}} = \frac{\sum_{j=1}^{N} w_j \left[D\left(\xi_{D,j}\right)L\left(\xi_{L,j}\right) + r\left(\xi_{r,j}\right)Q\left(\xi_{Q,j}\right)\right] \psi_{\boldsymbol{k}}^{S}(\xi_j)}{\left\|\psi_{\boldsymbol{k}}^{S}\right\|_{\boldsymbol{x}}^{2}}$$

Inner products

integrals

Gaussian quadrature

N **model evaluations**
Must not be too large
(tensorization!)
Contain stochastic variables!

Stochastic approach
Polynomial Chaos Expansion (PCE)
Intrusive approach (Galerkin)

$$S = S(\boldsymbol{\xi}) = \sum_{|\boldsymbol{k}|=0}^{P} c_{\boldsymbol{k}} \cdot \psi_{\boldsymbol{k}}^{S}(\boldsymbol{\xi})$$

In case of…

Differential system
$\mathcal{L}(S) = f$

Strong form

Weak form

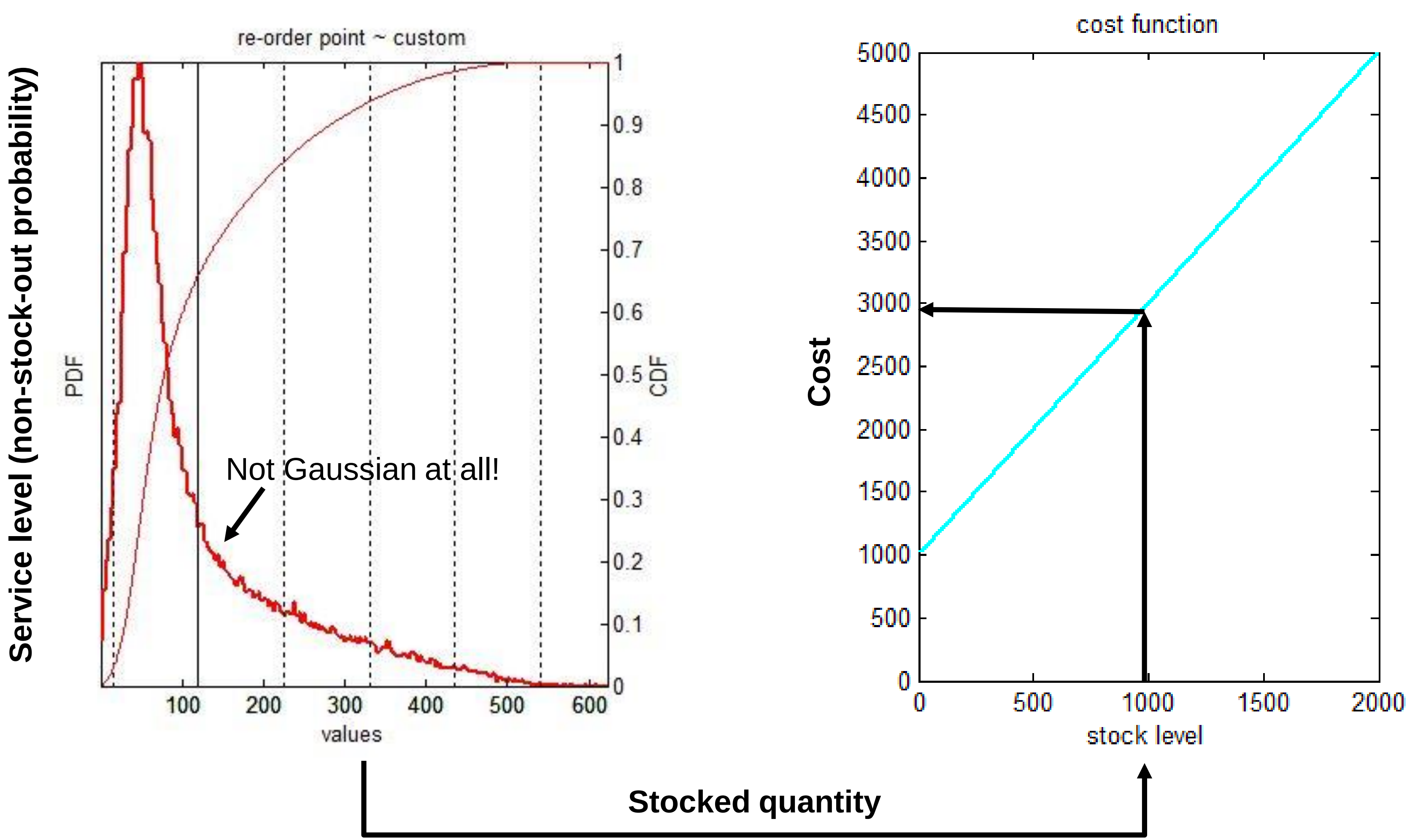
$$\left(\mathcal{L}(S), \psi_{\boldsymbol{j}}^{S}\right)_{\boldsymbol{x}} = \left(f, \psi_{\boldsymbol{j}}^{S}\right)_{\boldsymbol{x}}$$

$$\left(\mathcal{L}\left(\sum_{|\boldsymbol{k}|=0}^{P} c_{\boldsymbol{k}} \cdot \psi_{\boldsymbol{k}}^{S}\right), \psi_{\boldsymbol{j}}^{S}\right)_{\boldsymbol{x}} = \left(f, \psi_{\boldsymbol{j}}^{S}\right)_{\boldsymbol{x}}$$

In case of inear differential operators

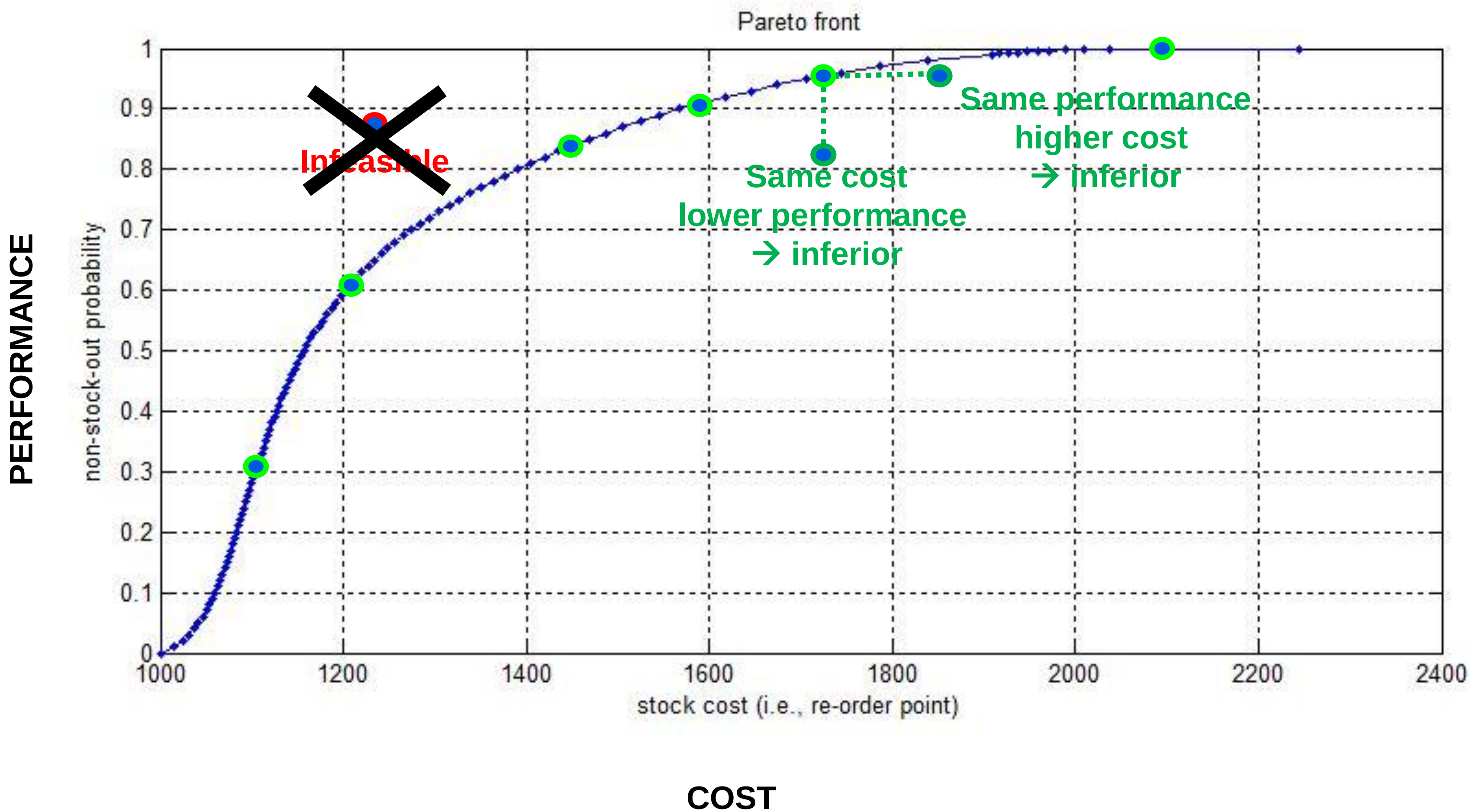
$$\sum_{|\boldsymbol{k}|=0}^{P} c_{\boldsymbol{k}} \cdot \left(\mathcal{L}\left(\psi_{\boldsymbol{k}}^{S}\right), \psi_{\boldsymbol{j}}^{S}\right)_{\boldsymbol{x}} = \left(f, \psi_{\boldsymbol{j}}^{S}\right)_{\boldsymbol{x}}$$

Stock sizing
Stochastic problem

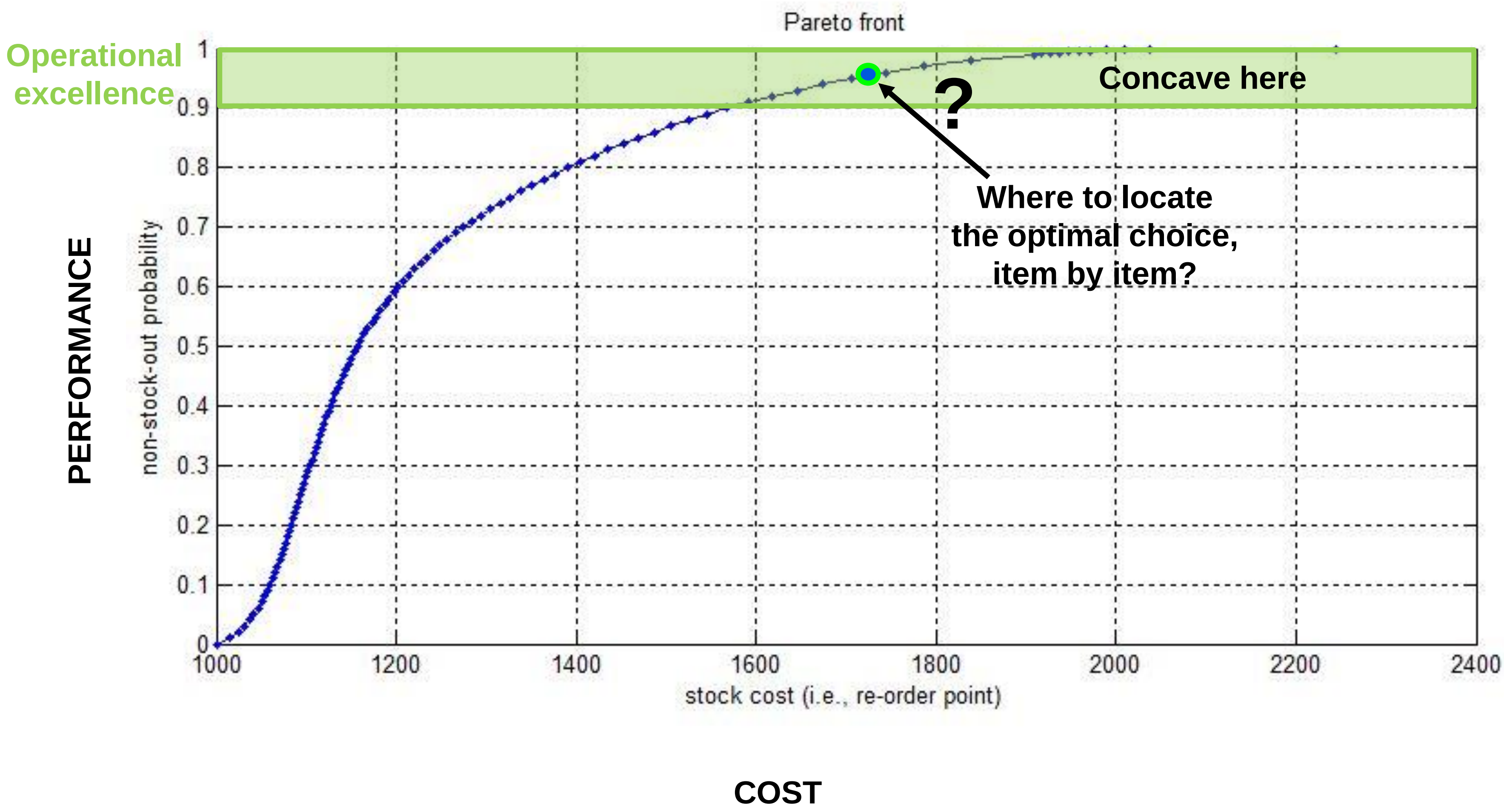


Stock sizing
Pareto Front, per single item

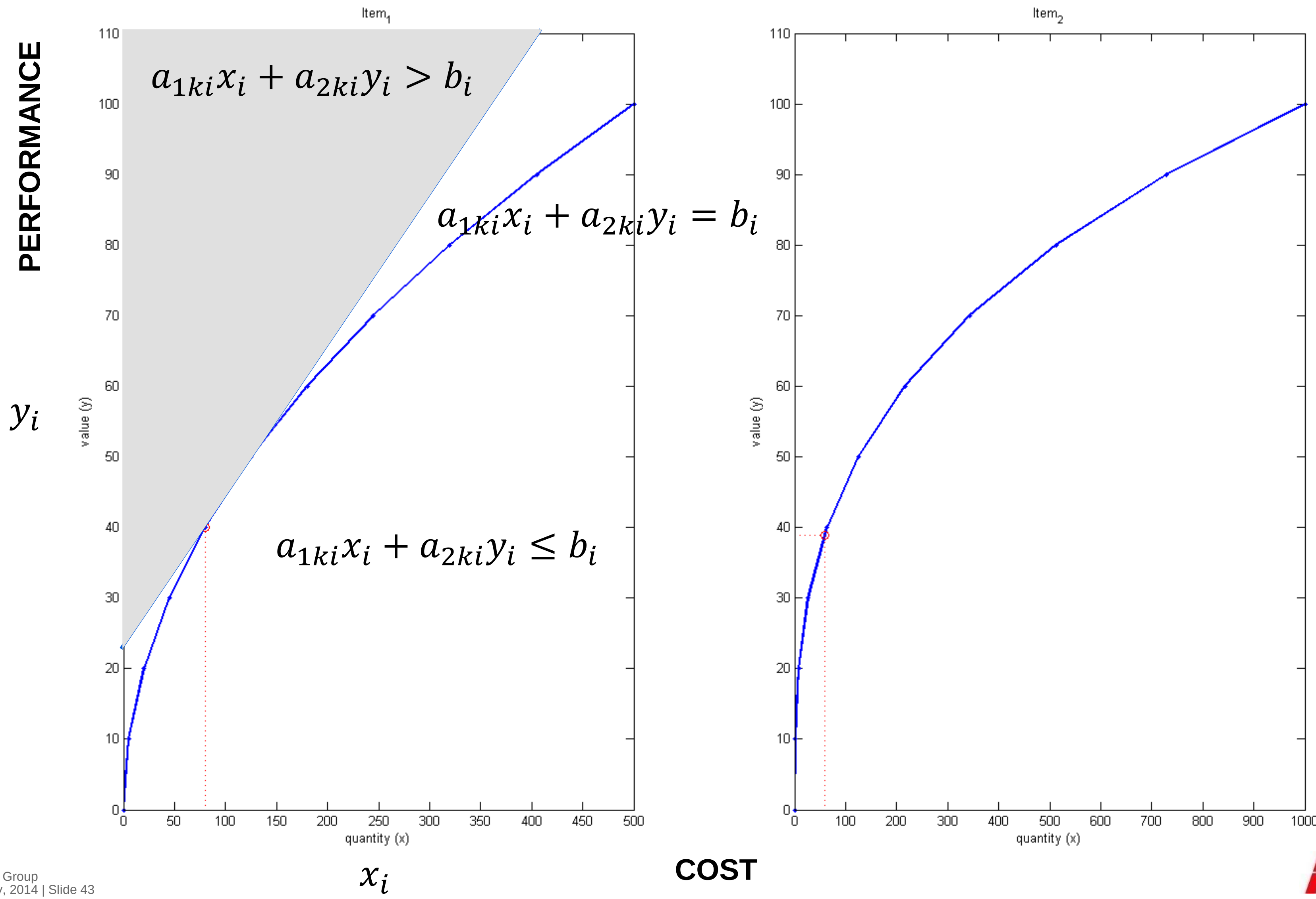
Each point on the front is «optimal» in the sense…



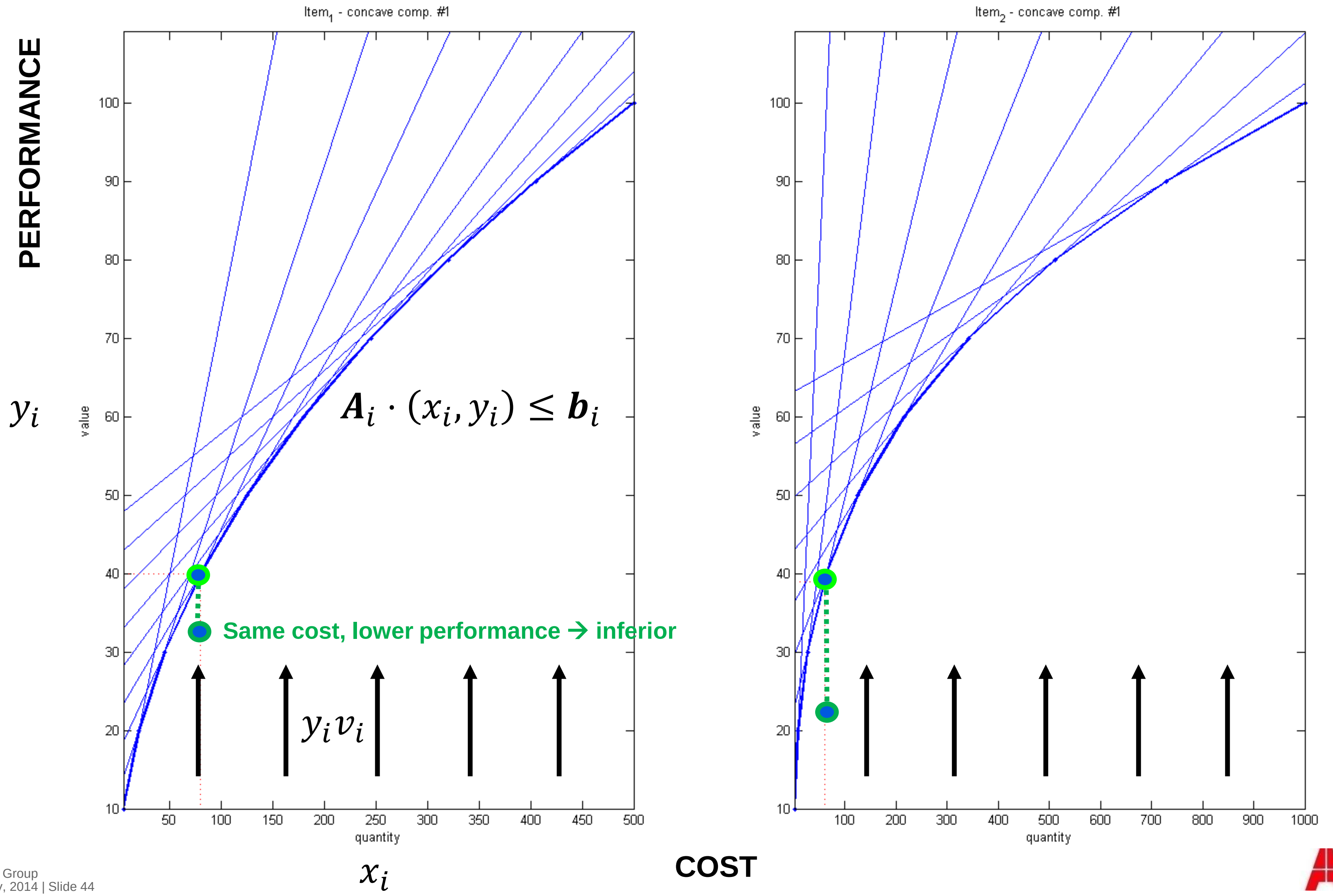
Stock sizing
Pareto Front, per single item



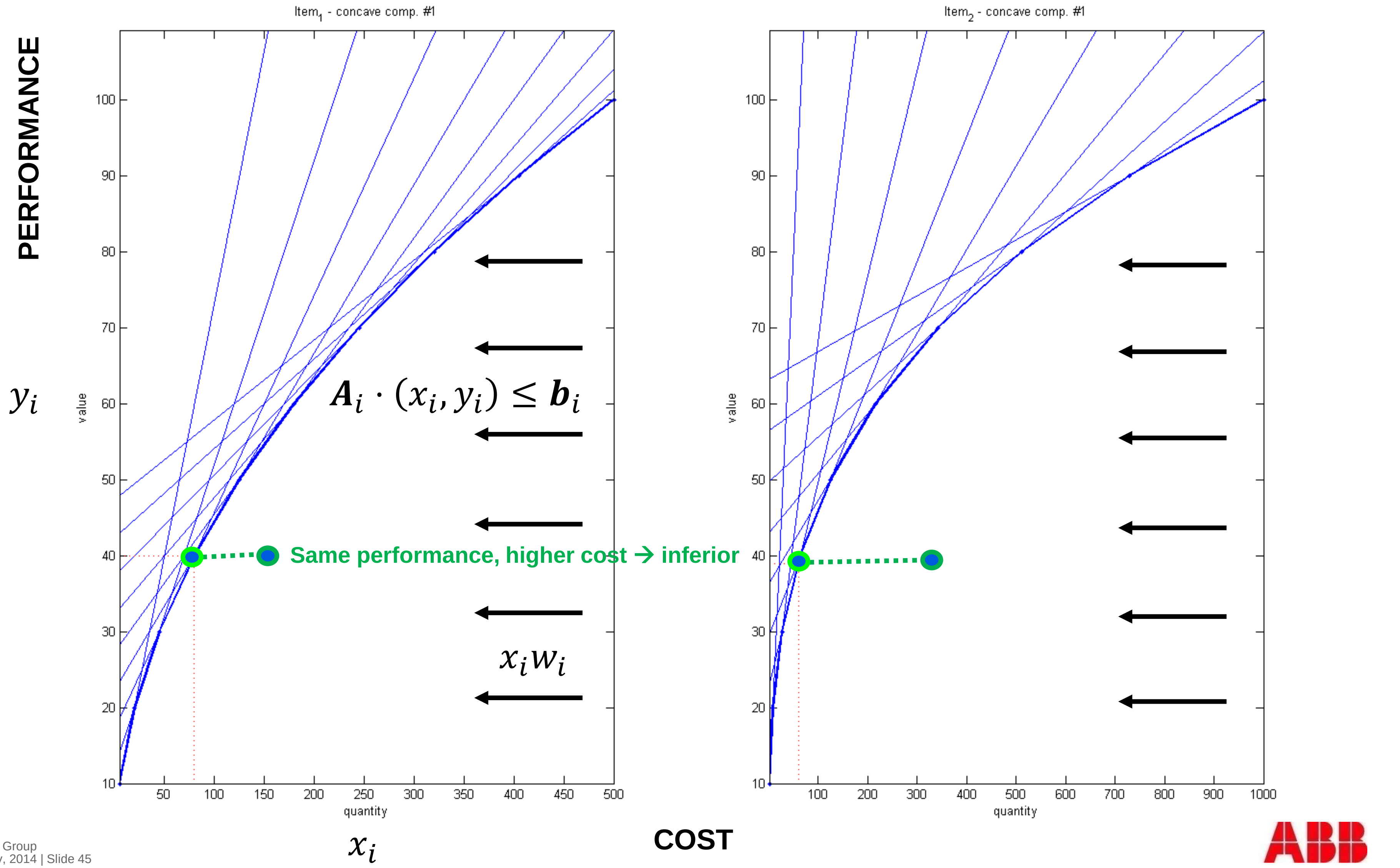
Stock sizing
Constrained optimization problem



Stock sizing
Constrained optimization problem



Stock sizing
Constrained optimization problem



Stock sizing
Constrained optimization problem

$$\max f = \sum_{i=1}^{n} y_i v_i$$

Goal function: performance

subject to

$$\sum_{i=1}^{n} x_i w_i \leq W,$$

Budget constraint

$$\boldsymbol{A}_i \cdot (x_i, y_i) \leq \boldsymbol{b}_i, \qquad \forall i \in \{1, \dots, n\}$$

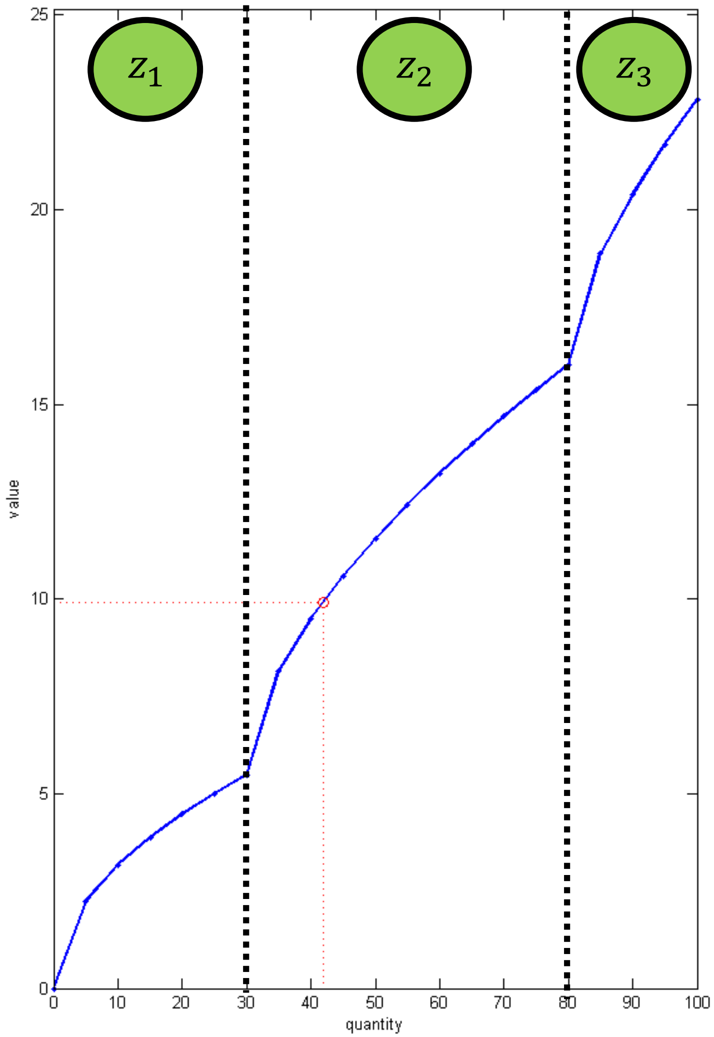
System laws constraint

$$l_i \leq x_i \leq u_i, \qquad \forall i \in \{1, \dots, n\}$$

Other possible bounds

- Nonlinear knapsack problem
- All variables real
- Many exact methods known (easy problem)
- Linear Programming (LP)
- Simplex method

Stock sizing
Constrained combinatorial optimization problem



- Non-concave case
- Made piecewise concave
- Dinsjunctive choice variables
  - Integer: $z_i \in \mathbb{Z}_2, i \in \{1,2,3\}$
  - $z_1 + z_2 + z_3 = 1$
  - Exactly one equals 1, others vanish
- Difficult problem

Stock sizing
Constrained combinatorial optimization problem

$$\max f = \sum_{i=1}^{n}(y_i v_i + z_i V_i - (1 - z_i) g_i(l_i) v_i)$$

subject to

$$\sum_{i=1}^{n}(x_i w_i + z_i W_i - (1 - z_i\,) l_i w_i) \leq W,$$

$$x_i - (u_i - l_i) z_i \leq l_i, \qquad \forall i \in \{1, \dots, n\}$$

$$\boldsymbol{A}_i \cdot (x_i, y_i) \leq \boldsymbol{b}_i, \qquad \forall i \in \{1, \dots, n\}$$

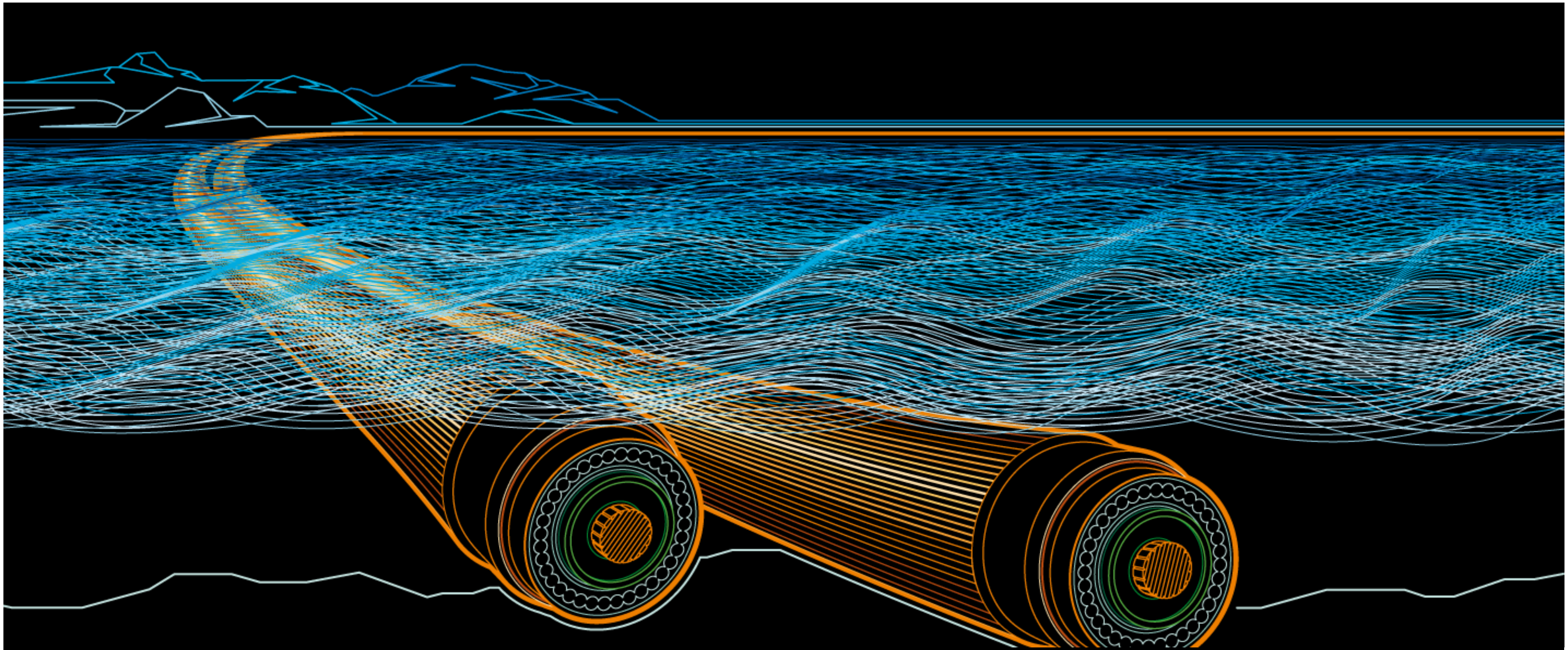
$$l_i \leq x_i \leq u_i, \qquad \forall i \in \{1, \dots, n\}$$

$$m h_i \leq z_i, \qquad \forall i \in \{1, \dots, n\,\}$$

$$z_i \in \{0,1\}, \qquad \forall i \in \{1, \dots, n\}$$

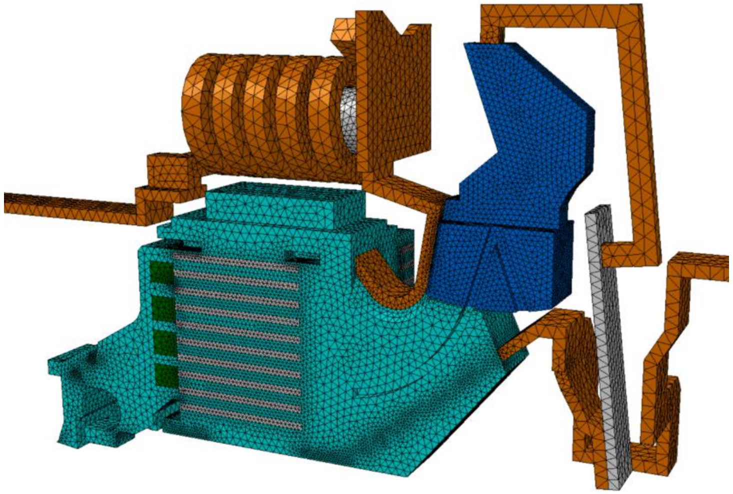
$$\sum_{j \in K_i} z_j \leq 1\,,$$ Dependency chain of non «must-have» items

$$\sum_{j \in K_i} z_j = 1\,,$$ Dependency chain of «must-have» items



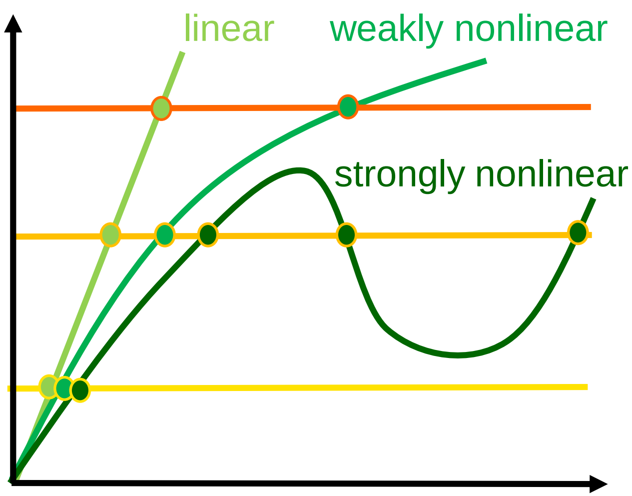
Future Research Trends

Future research trends
Increasing size



- **Industrial needs**
  - Large scale problems
  - Multi-processor HW is very cheap
  - Only marginal improvements to current transistor technology
- **Mathematical challenges**
  - Efficient and robust preconditioners
  - Parallel algorithms
  - Hierarchy and multi-level attacks

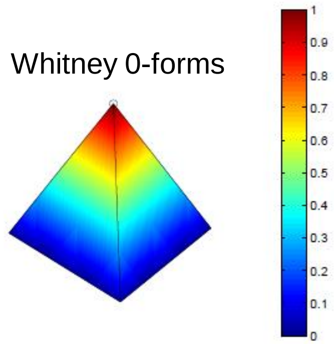
Future research trends
Increasing complexity



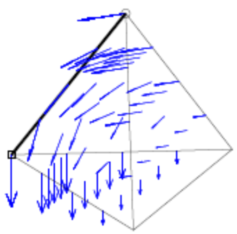
- **Industrial needs**
  - Nonlinear problems
  - Including strongly nonlinear!
- **Mathematical challenges**
  - Robust formulations
  - Necessary and/or sufficient conditions for convergence

Future research trends
Increasing specificity

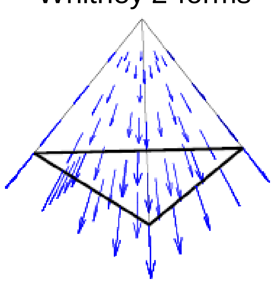
Whitney 0-forms



Whitney 1-forms



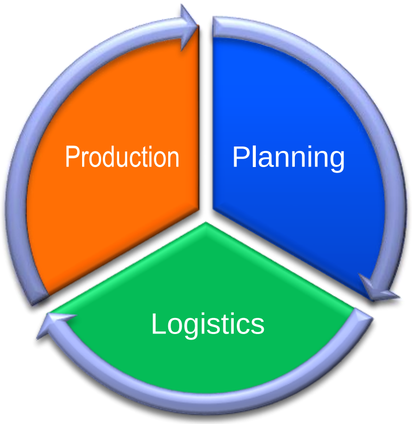
Whitney 2-forms



Whitney 3-forms = FVM

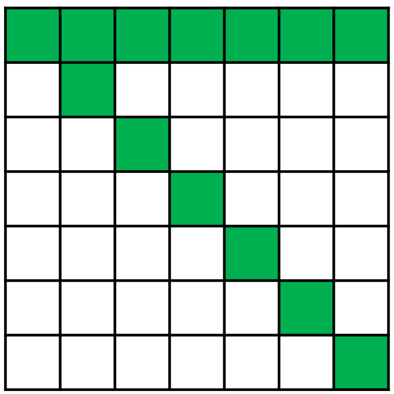
- **Industrial needs**
  - From general purpose to specific technologies
  - From general, «textbook» problems & solutions
  - To specific, taylorized problems & solutions
- **Mathematical challenges**
  - Exploit the mathematical structure of the operators
    - E.g., Whitney forms & algebraic topology
      → geometrically conformal comput. electromag.
  - Find out closest standard problem
  - Develop original approach to add specific features
    - Standard ideas exist, a lot of work to do
    - Innovative ideas welcome!

Future research trends
Integration of systems



- **Industrial needs**
  - Multi-phisical problems
  - Multi-scale problems
  - PDE’s + DAE’s
  - Multi-disciplinary combinatorial optimization
- **Mathematical challenges**
  - Robust formulations
  - Necessary and/or sufficient conditions for convergence
  - From weak coupling to strong coupling?
  - Original attacks to nonstandard discrete problems

Future research trends
Stochastics



- **Industrial needs**
  - Stochastic differential equations (SDE)
  - Non-deterministic problems and/or boundary conditions
  - Either intrinsically (finance, markets, etc.)
  - Or *de facto* (e.g., deterministic PDE with non completely known material properties or forcing terms)
- **Mathematical challenges**
  - Beyond MonteCarlo & non-intrusive, black-box attacks
  - «Stochastic dimensions» added to physical dimensions
    - Preconditioners, robustness, convergence, etc.
    - Pure mathematics as well!
  - «Large» number (> 10) of stochastic variables

Future research trends
Combinatorics

```
if z==1 then
    x >= LB;
else
    x = 0;
end
```

$$LB \cdot z \leq x \leq M \cdot z$$

$$(M \gg LB)$$

- **Industrial needs**
  - Discrete, combinatorial optimization (→ processes)
  - Sub-optimal solutions frequently adequate
    - A good solution to a realistic problem is better…
    - …than the mathematically best solution to an oversimplistic problem
- **Mathematical challenges**
  - Large scale
  - From pure (meta-)heuristics
  - To «ancillary» heuristics in exact methods
    - As a fast albeit sharp bounding tool
    - Branching + Bounding / Pricing / …

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for a better world™