ERC Workshop on

Energy/Entropy-Driven Systems and Applications

Weierstrass Institute for Applied Analysis and Stochastics
October 9–11, 2013

www.wias-berlin.de/workshops/EEDSA13

WIAS

Weierstrass Institute for
Applied Analysis and Stochastics

www.wias-berlin.de

erc

European Research Council
Established by the European Commission

erc.europa.eu

ERC Workshop on

Energy/Entropy-Driven Systems and Applications

Weierstrass Institute for Applied Analysis and Stochastics, Berlin, Germany
October 9–11, 2013

Organized by

Karoline Disser
Alexander Mielke
Ulisse Stefanelli

Supported by

European Research Council
Established by the European Commission

ERC project: Analysis of Multiscale Systems Driven by Functionals

Weierstrass Institute for Applied Analysis and Stochastics

Wednesday, October 9, 2013, 08:00 - 18:00

08:00 - 08:55	REGISTRATION
08:55 - 09:00	OPENING
09:00 - 09:45	Local minimization, variational evolution and Γ-convergence **Andrea Braides**
09:50 - 10:35	Two-scale decomposition for metastable dynamics in continuous and discrete setting **André Schlichting**
10:40 - 11:10	COFFEE BREAK
11:10 - 11:35	Gradient flows and reversible large deviations **Michiel Renger**
11:40 - 12:25	On gradient structures for reversible Markov chains and the passage to Wasserstein gradient flows **Matthias Liero**
12:30 - 14:30	LUNCH
14:30 - 15:15	Entropic flows, stochastic perturbations and microscopic models **Johannes Zimmer**
15:20 - 15:45	Inequalities for Markov operators and the direction of time **Holger Stephan**
15:50 - 16:20	COFFEE BREAK
16:20 - 17:05	Convergence rates of order one perturbations in the 1-d Cahn Hilliard equation **Maria G. Westdickenberg**
17:10 - 17:55	Convex Lyapunov functionals for non-convex gradient flows: two examples **Daniel Matthes**

Thursday, October 10, 2013, 09:00 - 18:10

09:00 - 09:45	Elliptic regularization of gradient flows in metric spaces **Riccarda Rossi**
09:50 - 10:35	Macroscopic interface motion in discrete forward-backward diffusion equations **Michael Herrmann**
10:40 - 11:10	COFFEE BREAK
11:10 - 11:35	A finite volume method for reaction-diffusion systems **André Fiebach**
11:40 - 12:25	Entropy-dissipative discretizations of nonlinear diffusive equations **Ansgar Jüngel**
12:30 - 14:30	LUNCH
14:30 - 15:15	Balanced Viscosity (BV) solutions of rate-independent systems **Giuseppe Savaré**
15:20 - 15:45	Optimality Conditions for Control Problems with Rate-Independent Elements **Martin Brokate**
15:50 - 16:20	COFFEE BREAK
16:20 - 17:05	Quasistatic evolution models for thin structures in perfect plasticity **Maria Giovanna Mora**
17:10 - 17:35	Scaling in fracture mechanics: from finite to linearized elasticity **Rodica Toader**
17:40 - 18:05	Quasistatic evolution models for thin plates arising as low energy Gamma-limits of finite plasticity **Elisa Davoli**
from 19:00	DINNER

Friday, October 11, 2013, 09:00 - 16:00

09:00 - 09:45	Stability of stationary states for repulsive-attractive potentials **José Antonio Carrillo**
09:45 - 10:30	Low temperature states in one-dimensional Lennard-Jones systems **Florian Theil**
10:30 - 10:50	COFFEE BREAK
10:50 - 11:15	Thermodynamic forces in single crystals with dislocations **Nicolas Van Goethem**
11:15 - 12:00	Energy dissipation and well-posedness for some problems in fluid dynamics **Eduard Feireisl**
12:00 - 12:45	On a non-isothermal diffuse interface model for two-phase flows of incompressible fluids **Elisabetta Rocca**
12:45 - 13:00	CLOSING
13:00	LUNCH

Local minimization, variational evolution and Γ-convergence

Andrea Braides

Università di Roma 'Tor Vergata', Italy
(on leave at the University of Oxford, UK, until July 2014)

The theory of Γ-convergence is by now commonly used as a means for describing multiscale phenomena. The main issue in its definition is tracking the behaviour of global minimum problems (minimum values and minimizers) of a sequence F_ε by the computation of an effective minimum problem involving the Γ-limit of this sequence. Even though the definition of such a limit is local (in that in defining its value at a point x we only take into account sequences converging to x), its computation in general does not describe the behaviour of local minimizers of F_ε (i.e., points x_ε are absolute minimizers of the restriction of F_ε to a small neighbourhood of the point x_ε itself). In general, some or all the local minimizers of F_ε may be 'integrated out' in the Γ-convergence process. A notable exception is when we have an isolated local minimizer x of the Γ-limit: in that case we may conclude the existence of local minimizers for F_ε close to x. The possibility of the actual application of such a general principle has been envisaged by Kohn and Sternberg [5], who first used it to deduce the existence of local minimizers of the Allen-Cahn equation by exhibiting local area minimizing sets. A recent different but related direction of research concerns the study of gradient flows. A general variational theory based on the solution of Euler schemes has been developed by Ambrosio, Gigli and Savaré [1]. The stability of such schemes by Γ-perturbations is possible in the absence of local minimizers which could generate pinned flows (i.e., stationary solutions or solutions 'attracted' by a local minimum) that are not detected by the limit (in constast with the theory of rate-independent evolution; see [8, 7]). Conditions that guarantee stability are of convexity type on the energies. These conditions can be removed under other special assumptions on the gradient flows and for 'well-prepared' initial data following the scheme proposed for Ginzburg-Landau energies by Sandier and Serfaty [9]. Unfortunately, as remarked by those authors, the applicability of this scheme is often hard to verify. Taking the above-mentioned results as a starting point we have explored some different directions. The standpoint of the analysis is that even though the Γ-limit may not give the correct description of the effect of local minimizers, it may nevertheless be 'corrected' in some systematic way.

A first issue beyond global minimization takes into account the notion of equivalence by Γ-convergence as introduced and studied by Braides and Truskinovsky [2]: in the case that a Γ-limit or a Γ-development may be insufficient to capture some desired features of the minimum problems of F_ε we may introduce equivalent energies G_ε. These energies still integrate out the unimportant details of F_ε but maintain the desired features and are equivalent to the original F_ε in that they have the same Γ-limit or Γ-development. One of the conditions that may be required to G_ε is that they have the same landscape of local minimizers as F_ε. As an example, a Γ-development taking into account interactions between neighbouring transitions recovers the local minimizers of Allen-Cahn energies that are integrated out by the usual sharp-interface models of phase transitions. Another issue is the problem of distinguishing 'meaningful local minimizers' from those that may 'rightfully' be considered to disappear in the limit. Local minimizers deriving from vanishing oscillations should be considered as different from those 'trapped' by energy barriers. To that end we may use the notion of δ-stable state as recently introduced by C. Larsen [6], and the related notion of stable sequences of energies [4]. We remark that Γ-convergence allows to exhibit classes of stable sequences.

Linked to the study of local minimizers is the variational motion defined by the limit of Euler schemes at vanishing time step. This motion has been usually defined for a single functional F (and is sometimes referred to as a minimizing movement). We have examined a variation of the minimizing-movement scheme with two parameters: one is the time step τ, and the second one is the parameter ε (that we may ofter regard as a space scale) appearing in the Γ-converging sequence F_ε .

The Euler scheme is then applied at fixed τ with $\varepsilon = \varepsilon(\tau)$, so that the resulting discrete trajectories (u_ε) may depend on the interaction between the two scales, and hence also their limits (the minimizing movements along the sequence F_ε). A general result, directly derived from the properties of Γ-convergence allows to deduce the existence of a 'fast' space scale such that the limit of the ε-τ Euler scheme is just a minimizing movement for the Γ-limit. For 'slow' space scales the motion is often 'pinned' at local minimizers. This observation highlights the existence of one or more critical ε-τ regimes which capture the most interesting features of the motion connected to these energies. Examples show that we may have cases where an 'effective' motion can be described such that all minimizing movements are obtained from that one by scaling. Another possible phenomenon for these Euler schemes for Γ-converging energies is the existence of more superposed time scales, whose motions can themselves be interpreted as derived from Euler schemes for scaled functionals $F_\varepsilon/\lambda_\varepsilon$. Moreover, an appropriate choice of Γ-approximating sequences to a given F may be used to define a 'backward' motion.

The results are collected in a forthcoming book [3].

REFERENCES

[1] L. Ambrosio, N. Gigli, G. Savaré, Gradient flows in metric spaces and in the space of probability measures. Lectures in Mathematics ETH Zürich, Birkhhäuser, Basel, 2008.

[2] A. Braides, L. Truskinovsky, Asymptotic expansions by Gamma-convergence. Cont. Mech. Therm. **20** (2008), 21-62.

[3] A. Braides, Local Minimization, Variational Evolution and Γ-convergence. Lecture Notes in Mathematics **2094**, Springer, Berlin, 2013.

[4] A. Braides, C.J. Larsen, Γ-convergence for stable states and local minimizers. Ann. Scuola Norm. Sup. Pisa **10** (2011), 193-206.

[5] R.V. Kohn, P. Sternberg, Local minimizers and singular perturbations. Proc. Roy. Soc. Edinburgh A **111** (1989), 69-84.

[6] C.J. Larsen, Epsilon-stable quasi-static brittle fracture evolution. Comm. Pure Appl. Math. **63** (2010), 630–654.

[7] A. Mielke, Evolution of Rate-Independent Systems. In: Dafermos, C.M., Feireisl E. (eds) Handbook of Partial Differential Equations. Evolutionary Equations, Volume **2**, 461–560, Elsevier, Amsterdam, 2005.

[8] A. Mielke, T. Roubiček, U. Stefanelli, Γ-limits and relaxations for rate-independent evolutionary problems. Calc. Var. Part. Diff. Equ. **31** (2008), 387–416.

[9] E. Sandier, S. Serfaty, Gamma-Convergence of Gradient Flows and Application to Ginzburg-Landau, Comm. Pure Appl. Math. **57** (2004), 1627–1672.

Optimality conditions for control problems with rate-independent elements

Martin Brokate

Zentrum Mathematik, TU München, Germany

We present first order optimality conditions for the optimal control of a time-dependent system coupled to a rate independent evolution variational inequality.

Stability of stationary states for repulsive-attractive potentials

José Antonio Carrillo

Department of Mathematics, Imperial College London, UK

I will discuss nonlocal integral continuity equations for which notrivial stationary states show interesting properties and patterns as shown numerically. They are first order minimal models for swarming. We will discuss their nonlinear stability/instability and the dimension of their support. They lead to interesting questions of calculus of variations in measure settings.

Quasistatic evolution models for thin plates arising as low energy Gamma-limits of finite plasticity

Elisa Davoli

Dept. of Mathematical Sciences, Carnegie Mellon University, Pittsburgh (PA), USA

In this talk we shall deduce by Γ-convergence some partially and fully linearized quasistatic evolution models for thin plates, in the framework of finite plasticity. Denoting by ϵ the thickness of the plate, we study the case where the scaling factor of the elasto- plastic energy is of order $\epsilon^{2\alpha-2}$, with $\alpha \geq 3$. We show that solutions to the three- dimensional quasistatic evolution problems converge, as the thickness of the plate tends to zero, to a quasistatic evolution associated to a suitable reduced model depending on α.

REFERENCES

[1] E. Davoli, Linearized plastic plate models as Γ-limits of 3D finite elastoplasticity. Preprint SISSA, 2012.

[2] E. Davoli, Quasistatic evolution models for thin plates arising as low energy Γ-limits of finite plasticity. Preprint SISSA, 2012.

A finite volume method for reaction-diffusion systems

André Fiebach, Annegret Glitzky, Klaus Gärtner, Alexander Linke

Weierstrass Institute, Berlin, Germany

For reversible chemical reactions of mass action type with diffusion, heterogeneous materials, and Neumann boundary conditions in two space dimensions a Voronoi finite volume scheme is introduced. It preserves the known analytic properties [1]: existence of a unique thermodynamic equilibrium solution, exponential decay of the free energy along trajectories towards thermodynamic equilibrium, existence of global bounded positive solutions and convergence to the weak solution.

The proofs are based on energy estimates, mass preservation, Moser-iteration, and L^{∞}-bounds, [2, 3, 4]. The method is illustrated by a few examples using the Michaelis-Menten-Henri kinetics with strongly varying timescales. The heterogeneous materials can cause strong boundary layers. The numerical algorithm is preserving the two integral mass invariants of the kinetics precisely. The diffusion and reaction coefficients can be chosen such that the thermodynamic equilibrium is separated fifty orders of magnitude in time away from the initial value.

REFERENCES

[1] H. Gajewski, K. Gröger, Reaction – Diffusion Processes of Electrically Charged Species. Math. Nachr. **177**(1) (1996), 109–130.

[2] A. Glitzky, Uniform exponential decay of the free energy for Voronoi finite volume discretized reaction-diffusion systems. Math. Nachr. **284**(17-18) (2011), 2159–2174.

[3] A. Fiebach, A. Glitzky, A. Linke, Uniform global bounds for solutions of an implicit Voronoi finite volume method for reaction-diffusion problems. WIAS preprint 1718, 2012.

[4] A. Fiebach, A. Glitzky, A. Linke, Convergence of an implicit Voronoi finite volume method for reaction-diffusion problems. WIAS preprint 1827, 2013.

Energy dissipation and well-posedness for some problems in fluid dynamics

Eduard Feireisl

Institue of Mathematics, Academy of Sciencs of the Czech Republic, Prague

We discuss the implications of the second law of thermodynamics on the problem of well posedness in the class of weak solutions for certain problems in fluid dynamics. We introduce the concept of dissipative solutions and show the principle of weak-strong uniqueness in the class of viscous and inviscid fluids. Then we discuss the criterion of maximal dissipation for the inviscid compressible Euler system in the context of the solutions obtained by the method of convex integration.

Macroscopic interface motion in discrete forward-backward diffusion equations

Michael Helmers[(1)] and Michael Herrmann[(2)]

(1) University of Bonn, Germany
(2) Saarland University, Germany

We study the propagation of phase interfaces in the discrete diffusion equation

$$\dot{u}_j = \Delta p_j \qquad \text{with} \qquad p_j = \Phi'\left(u_j\right),$$

where Δ denotes the discrete Laplacian and Φ' is the bistable derivative of a double-well potential Φ. Such lattice systems can be regarded as spatial regularizations of certain ill-posed parabolic gradient flows and admit several types of moving or standing phase interfaces.

In the first part we allow for generic bistable non-linearities and combine heuristic arguments with numerical evidence to describe the effective dynamics in the parabolic scaling limit by a nonlinear PDE with hysteresis operator. In particular, the reduced model for the lattice is the same as for the viscous approximation but differs from the limit model for the Cahn-Hilliard equation, see [1, 2, 3].

In the second part we specialize to a piecewise affine nonlinearity and derive the hysteretic limit model rigorously from the lattice dynamics. To this end we prove persistence of single-interface data, establish upper bounds for the macroscopic interface speed, and derive compactness results in suitable function spaces. The key ingredient to our proof is a summation formula that represents the solutions of the nonlinear lattice in terms of the discrete heat kernel.

References

[1] L. C. Evans and M. Portilheiro, Irreversibility and hysteresis for a forward-backward diffusion equation. Math. Models Methods Appl. Sci. **14**(11) (2004), 1599–1620.

[2] M. Helmers and M. Herrmann, Interface dynamics in discrete forward-backward diffusion equations. Submitted, 2013. `arXiv:1304.1693`.

[3] A. Visintin, Quasilinear parabolic P.D.E.s with discontinuous hysteresis. Ann. Mat. Pura Appl. **185**(4) (2006), 487–519.

Entropy-dissipative discretizations of nonlinear diffusive equations

Ansgar Jüngel

Institute for Analysis and Scientific Computing,
Vienna University of Technology, Austria

We present numerical discretizations which preserve the entropy structure of the analyzed nonlinear diffusive equations. More precisely, we develop numerical schemes for which the discrete entropy is stable or even dissipating. The key idea is to 'translate' entropy-dissipation methods to the discrete case. We consider two situations.

First, an implicit Euler finite-volume approximation of porous-medium or fast-diffusion equations is investigated [2]. The scheme dissipates all zeroth-order entropies which are dissipated by the continuous equation. The proof is based on novel discrete generalized Beckner inequalities. Furthermore, the exponential decay of some first-order entropies is proved using systematic integration by parts and a convexity property with respect to the time step parameter.

Second, new one-leg multistep time approximations of general nonlinear evolution equations are investigated [1, 3]. These schemes preserve both the nonnegativity and the entropy-dissipation structure of the equations. The key idea is to combine Dahlquist's G-stability theory with entropy-dissipation methods. The optimal second-order convergence rate is proved under a certain monotonicity assumption on the operator. The discretization is applied to a cross-diffusion system from population dynamics and a fourth-order quantum diffusion equation.

References

[1] M. Bukal, E. Emmrich, and A. Jüngel, Entropy-stable and entropy-dissipative approximations of a fourth-order quantum diffusion equation. Preprint, 2012. `arXiv:1208.5474.`

[2] C. Chainais-Hillairet, A. Jüngel, and S. Schuchnigg, Entropy-dissipative discretization of nonlinear diffusion equations and discrete Beckner inequalities. Preprint, 2013. `arXiv:1303.3791`.

[3] A, Jüngel and J. P. Milišić, Entropy dissipative one-leg multistep time approximations of nonlinear diffusive equations. In preparation, 2013.

On gradient structures for reversible Markov chains and the passage to Wasserstein gradient flows

Karoline Disser(1), Matthias Liero(1)

(1) Weierstrass Institute, Berlin, Germany

In this talk we discuss the limit passage from reversible, time-continuous Markov chains to the one-dimensional Fokker–Planck equation with linear drift. In [Mie11] and [Maa11] it was shown that Markov chains satisfying the reversibility condition (also called detailed balance condition) have entropic gradient structures. More precisely, the evolution of a reversible Markov chain on the finite state space $\{1, \ldots, n\}$ and with intensity matrix $\mathbb{A}_n$ can be written as

$$\dot{u} = \mathbb{A}_n u = -\mathbb{K}_n(u)\mathrm{D}E_n(u). \tag{1}$$

Here, the driving functional E_n is the relative entropy and $\mathbb{K}_n(u) = \mathbb{G}_n(u)^{-1}$ denotes the state-dependent, symmetric, and positive semi-definite Onsager matrix, which is the inverse of the metric tensor $\mathbb{G}_n(u)$.

In particular, reversible Markov chains arise as finite volume discretizations of the Fokker–Planck equation. Using the entropy/entropy-dissipation formulation of (1) we show that solutions of (1) converge to a solution of the Wasserstein formulation of the Fokker–Planck equation when the fineness of the partitions goes to zero.

Here, we only use the gradient structures of the systems and prove a Γ-convergence result for the relative entropy and dissipation potentials. Finally, we address the question of a generalization to higher dimenions.

REFERENCES

[Maa11] J. Maas, Gradient flows of the entropy for finite Markov chains. J. Funct.Anal. **261**(8) (2011), 2250–2292.

[Mie11] A. Mielke, A gradient structure for reaction-diffusion systems and for energy-drift-diffusion systems. Nonlinearity **24** (2011), 1329–1346.

Price-formation modelling: from Boltzmann to free boundaries

Peter Markowich

Department of Applied Mathematics and Theoretical Physics (DAMTP), University of Cambridge, UK

We present a Boltzmann-type model for the evolution of buyer and vendor densities in an economic market. In the limit of large transaction numbers the mean field parabolic free boundary problem as established by Lasry and Lions is obtained, in analogy to large-Knudsen number limits in gas-kinetics.

Convex Lyapunov functionals for non-convex gradient flows: two examples

Daniel Matthes

Zentrum Mathematik, TU München, Germany

We discuss two examples of gradient flows for non-convex potentials in the Wasserstein metric. To prove qualitative properties of weak solutions obtained by means of the JKO scheme, we use geodesically convex auxiliary functionals that produce meaningful dissipations.

The first example (joint work with J.Zinsl) is a system of two nonlinear diffusion equations modeling the aggregation of bacteria. Here we convexify the energy and use the resulting dissipation to prove exponential convergence of weak solutions to equilibrium. The second example (joint work with M.DiFrancesco) is a gradient flow that is equivalent to a nonlinear conservation law by a change of variables. By using appropriate auxiliary functionals, we prove that the weak solutions obtained from JKO are actually entropy solutions, which shows a posteriori their uniqueness.

Quasistatic evolution models for thin structures in perfect plasticity

Maria Giovanna Mora

Università di Pavia, Italy

In this talk I shall discuss the rigorous derivation of quasistatic evolution models for thin structures in the framework of Prandtl-Reuss plasticity. In the case of a thin plate such a limiting model has a genuinely three-dimensional nature, unless specific data are prescribed. In particular, the stretching and the bending components of the stress decouple only in the equilibrium condition, while the whole stress is involved in the stress constraint and in the flow rule.

I shall also present an *ad hoc* notion of stress-strain duality, which allows one to write a strong formulation of the flow rule in the limit problem.

This is based on the recent paper [1] and ongoing work with Elisa Davoli (Carnegie Mellon University).

REFERENCES

[1] E. Davoli, M.G. Mora, A quasistatic evolution model for perfectly plastic plates derived by Gamma-convergence. Ann. Inst. H. Poincaré Anal. Nonlin., to appear.

Gradient flows and reversible large deviations

Alexander Mielke[(1)], **Mark Peletier**[(2)], **and Michiel Renger**[(1)]

(1) Weierstrass Institute, Berlin, Germany
(2) University of Eindhoven, the Netherlands

Recent work reveals that a large class of entropy-Wasserstein-type gradient flows can be linked directly to stochastic particle systems via its large deviations [1, 2, 3, 4, 5, 6]. This suggests that there must be a general principle behind, which was so far still unknown. In this talk I present such a principle, and show how to construct a 'generalised gradient flow' for any reversible system.

References

[1] S. Adams, S., N. Dirr, M. A. Peletier, J. Zimmer, From a large-deviations principle to the Wasserstein gradient flow: a new micro-macro passage, Commun. in Math. Phys. **307** (2011), 791–815.

[2] –, Large deviations and gradient flows. Philosophical Transactions A, to appear. `http://arxiv.org/abs/1201.4601`.

[3] M. A. Peletier, D. R. M. Renger, M. Veneroni, Variational formulation of the Fokker-Planck equation with decay: a particle approach. Communications in Contemporary Mathematics, to appear. `http://arxiv.org/abs/1108.3181`.

[4] M. H. Duong, V. Laschos, D. R. M. Renger, Wasserstein gradient flows from large deviations of thermodynamic limits. ESAIM: COCV, to appear. `http://arxiv.org/abs/1203.0676v2` .

[5] M. H. Duong, M. A. Peletier, J. Zimmer, Conservative-dissipative approximation schemes for a generalized Kramers equation. Submitted, 2012. `http://arxiv.org/abs/1302.1024`.

[6] –, GENERIC formalism of a Vlasov-Fokker-Planck equation and connection to large-deviation principles. Submitted, 2013. `http://arxiv.org/abs/1302.1024` .

On a non-isothermal diffuse interface model for two-phase flows of incompressible fluids

Michela Eleuteri[(1)], **Elisabetta Rocca**[(2)], **and Giulio Schimperna**[(3)]

(1) University of Milan, Italy
(2) Weierstrass Institute, Berlin, Germany
(3) University of Pavia, Italy

We introduce a diffuse interface model describing the evolution of a mixture of two different viscous incompressible fluids of the same density. The effects of temperature on the flow are taken into account. In the mathematical model, the evolution of the velocity $\mathbf{u}$ is ruled by the Navier-Stokes system with temperature-dependent viscosity, while the order parameter φ representing the concentration of one of the components of the fluid is assumed to satisfy a convective Cahn-Hilliard equation. The effects of the temperature are prescribed by a suitable form of the heat equation. However, due to quadratic forcing terms, this equation is replaced, in the weak formulation of the model, by an equality representing energy conservation complemented with a differential inequality describing production of entropy. In this way, thermodynamical consistency is preserved, but the energy-entropy formulation is more tractable mathematically. Global-in-time existence for the initial-boundary value problem associated to the weak formulation of the model is proved by deriving suitable a-priori estimates and showing weak sequential stability of families of approximating solutions.

Elliptic regularization of gradient flows in metric spaces

Riccarda Rossi

University of Brescia, Italy

In this talk, based on an ongoing collaboration with Giuseppe Savaré, Antonio Segatti, and Ulisse Stefanelli, we address a variational approach to gradient-flow evolution in metric spaces. More precisely, we consider the WED (Weighted Energy Dissipation) functional, defined on trajectories: the Euler-Lagrange equation for its minimization can be considered as an elliptic regularization of the metric gradient flow. Our main result states that its minimizers converge to curves of maximal slope for geodesically convex energies. The crucial step of the argument is the reformulation of the variational approach in terms of a dynamic programming principle, and the use of the corresponding Hamilton-Jacobi equation. The result is applicable to a large class of nonlinear evolution PDEs having a gradient flow structure.

Balanced Viscosity (BV) solutions of rate-independent systems

Giuseppe Savaré

University of Pavia, Italy

Balanced Viscosity solutions to rate-independent systems arise as limits of regularized gradient flows obtained by adding a superlinear vanishing viscosity dissipation.

We address the main issues of the existence of such limits for infinite dimensional systems and of their characterization by a couple of variational properties combining a local stability condition and a balanced energy dissipation identity. We will also show that this notion provides a natural variational description of the jump behaviour of the solutions.

Our techniques rely on a suitable chain rule inequality for functions of bounded variation in Banach spaces, on refined lower semicontinuity-compactness arguments, and on new BV-estimates that are of independent interest.

(In collaboration with Alexander Mielke and Riccarda Rossi)

References

[1] A. Mielke, R. Rossi, G. Savaré, BV solutions and viscosity approximations of rate-independent systems. ESAIM: Control, Optimisation and Calculus of Variations **18** (2012), 36–80.

[2] –, Variational convergence of gradient flows and rate-independent evolutions in metric spaces. Milan J. Math. **80** (2012), 381–410.

[3] –, Nonsmooth analysis of doubly nonlinear evolution equations. Calc. Var. Partial Differential Equations **46** (2013), 253–310.

[4] –, Balanced Viscosity (BV) solutions to infinite-dimensional rate-independent systems. In preparation.

[5] R. Rossi, G. Savaré, A characterization of Energetic and BV solutions to one-dimensional rate-independent systems. Discrete Contin. Dyn. Syst. (S) **6** (2013), 167–191.

Two-scale decomposition for metastable dynamics in continuous and discrete setting

Georg Menz(1), **André Schlichting**(2), **and Martin Slowik**(3)

(1) Stanford University, USA
(2) University of Bonn, Germany
(3) Technische Universität Berlin, Germany

Metastable dynamics is characterized by the existence of at least two time scales on which the system shows different behavior. On the short time-scale, the system will reach some local equilibrium which is confined to some strict subset of the state space. Convergence to the equilibrium of the systems happens on the longest time scales which is characterized by rare transitions between these metastable states.

In the first part based on [5] and [6], we consider a diffusion on an energy landscape in the regime of low temperature which provides the metastable parameter. Metastability is manifested in the so-called Eyring-Kramers formula for the optimal constant in the Poincaré inequality (PI). The proof is based on a refinement of the two-scale approach introduced in [3] and of the mean-difference estimate introduced in [2]. In addition, this approach generalizes to the optimal constant in the logarithmic Sobolev inequality (LSI) for which potential theory [1] and semiclassical analysis [4] are not directly applicable.

The two main ingredients are good local PI and LSI constants, and sharp control of the mean difference between metastable regions. The first is obtained by constructing a Lyapunov function for the diffusion restricted to metastable regions. This mimics the fast convergence of the diffusion to metastable states. The mean-difference is estimated by a transport representation of weighted negative Sobolev-norms. It contains the main contribution to the PI and LSI constant, resulting from exponentially long waiting times of jumps between metastable states of the diffusion.

The last part [7] consists of an outlook of how these ideas carry over to reversible Markov chains. We use the link to the potential theoretic approach and consider metastable Markov chains in the sense of [1]. The metastable parameter could also be, besides temperature, the system size. This provides a two scale decomposition into metastable states. The main ingredient in this setting is the use of negative Sobolev norms in discrete setting together with its variational characterization and its relation to capacities.

References

[1] A. Bovier, Metastability in Methods of Contemporary Statistical Mechanics. R. Kotecky (Ed.), LNM **1970** (2009), Springer.

[2] D. Chafaï,F. Malrieu, On fine properties of mixtures with respect to concentration of measure and Sobolev type inequalities. Ann. Inst. Henri Poincaré Probab. Stat. **46**(1) (2010), 72–96.

[3] N. Grunewald, F. Otto, C. Villani, M. G. Westdickenberg, A two-scale approach to logarithmic Sobolev inequalities and the hydrodynamic limit. Ann. Inst. Henri Poincaré Probab. Stat. **45**(2) (2009), 302–351.

[4] B. Helffer, M. Klein, F. Nier, Quantitative analysis of metastability in reversible diffusion processes via a Witten complex approach. Mat. Contemp. **26** (2004), 41–85.

[5] G. Menz, A. Schlichting, Poincaré and logarithmic Sobolev inequalities by decomposition of the energy landscape. 2013. `arXiv:1202.1510`.

[6] A. Schlichting, The Eyring-Kramers formula for Poincaré and logarithmic Sobolev inequalities. PhD thesis, University of Leipzig, 2012.

[7] A. Schlichting and M. Slowik, Capacities, eigenvalues and logarithmic Sobolev constants for metastable Markov chains. Work in progress, 2013.

[8] M. Slowik. Contribution to the potential theoretic approach with applications to random field Curie-Weiss-Potts models. PhD thesis, Technische Universität Berlin, 2012.

Inequalities for Markov operators and the direction of time

Holger Stephan

Weierstrass Institute, Berlin, Germany

Relative entropy inequalities are well-known as a tool of investigation of the time behavior of the solutions to evolution equations like the Fokker-Planck equation. These inequalities show time irreversibility of the equations. This fact is usually understood as an expression of the second law of thermodynamics for isolated systems.

The solution operators of the mentioned equations are special cases of adjoints of Markov operator, i.e., operators conserving positivity and mass. Markov operators provide large classes of inequalities based on Jensen's inequality for convex as well as for special types of non-convex functions [1, 2].

Considering Markov operators in a general mathematical setting [3], applicable for general classical physical systems, it becomes clear that time irreversibility holds not only for isolated systems. In addition, the inequalities show the irreversibility of time for both forward as well as backward-running time, depending on the physical meaning of the variables in the underlying equations. Moreover, the order induced by the mentioned inequalities generalizes the order in majorization theory well-known from linear algebra and integration theory.

References

[1] C. Niculescu, H. Stephan, A generalization of Lagrange's algebraic identity and connections with Jensen's inequality. WIAS preprint 1756, 2012.

[2] H. Stephan, Lyapunov functions for positive linear evolution problems. ZAMM Z. Angew. Math. Mech. **85** (2005), 766-777.

[3] H. Stephan, A mathematical framework for general classical systems and time irreversibility as its consequence. WIAS preprint 1629, 2011.

Scaling in fracture mechanics: from finite to linearized elasticity

Rodica Toader

University of Udine, Italy

The propagation of cracks in elastic solids was studied, since the very beginning, within the framework of linearized elasticity. This approach seems questionable since, one century ago, in 1913, Inglis [2] proved that in an (ideal) infinite linear elastic solid the stress around elliptical holes and cracks is proportional to the inverse of the square of the radius of curvature. His result seems to suggest that in the presence of a large curvature of the boundary the linearized elasticity might not represent the correct framework since strain and stress are very large (infinite in the case of a crack). Nevertheless Linear Elastic Fracture Mechanics has been successfully employed in realistic applications and this leads to think that the effect of the nonlinear setting on the propagation of cracks should often be negligible.

In this contribution I will comment on the relationship between finite and linearized elasticity in fracture mechanics, specifically in the case of quasi-static crack evolutions in brittle bodies. In elasto-statics, this relationship was analysed in [1] where the convergence of the rescaled nonlinear energies towards the linearized energy was shown, as well as the convergence of the corresponding minimizers. This result and its later refinements settle the static case. As fracture mechanics is concerned, the relationship between linear and nonlinear energies was studied in [5] assuming that the crack evolves along a prescribed segment. Here I will focus on a result obtained in a joint work with Matteo Negri [4], on the derivation of a quasi-static crack evolution in a brittle body in the framework of linear elastic fracture mechanics from finite elasticity quasi-static crack evolutions on suitably rescaled domains. The main issue in this contribution is that the path followed by the crack is not prescribed; it is determined by the minimization process among curves in a suitable class of admissible cracks introduced in [3]. As a fundamental tool we use the Γ-convergence of the rescaled finite elasticity energies to the linearized energy. The main advantage is that Γ-convergence guarantees the convergence of the corresponding minimizers. The scaling we adopt is the same suggested by Bažant's law and confirmed by experiments. Our analysis shows that the whole evolution converges while experimental results typically show convergence of the nominal strength.

References

[1] G. Dal Maso, M. Negri, D. Percivale, Linearized elasticity as Γ-limit of finite elasticity. Set-Valued Anal. **10** (2002), 165–183.
[2] C.E. Inglis, Stress in a plate due to the presence of sharp corners and cracks. Trans. Roy. Inst. Naval Architects **60** (1913), 219–241.
[3] G. Lazzaroni, R. Toader, Energy release rate and stress intensity factor in antiplane elasticity. J. Math. Pures Appl. **95** (2011), 565–584.
[4] M. Negri, R. Toader, Scaling in fracture mechanics by Bažant's law: from finite to linearized elasticity. In preparation.
[5] M. Negri, C. Zanini, From finite to linear fracture mechanics by scaling. Preprint CVGMT, 2012.

Thermodynamic forces in single crystals with dislocations

Nicolas Van Goethem

SISSA, Trieste, Italy

A striking geometric property of elastic bodies with dislocations is their non-Riemannian nature and it turns out that the nonvanishing torsion associated with the elastic metric is a physical field related to the dislocation density tensor. On the other hand the Riemann curvature tensor is also related to strain incompatibility, itself provoked by the presence of dislocations.

In this talk, a simple model for the evolution of macroscopic dislocation regions in a single crystal will be presented. This model relies on maximal dissipation principle within Kröner's geometric description of the dislocated crystal [1, 4]. Mathematical methods and tools from shape optimization theory provide equilibrium relations at the dislocation front, similarly to previous work achieved on damage modelling [2]. The deformation state variable is the incompatible strain as related to the dislocation density tensor by a relation involving the Ricci curvature of the crystal underlying elastic metric. The time evolution of the model variables follows from a novel interpretation of the Einstein-Hilbert flow [3] in terms of dislocation microstructure energy. This flow is interpreted as the dissipation of non-conservative dislocations due to the climb mechanism, modelled by an average effect of mesoscopic dislocations moving normal to their glide planes by adding or removing points defects. The model equations are a 4th-order tensor parabolic equation involving the operator "incompatibility", here appearing as a nonstandard tensorial counterpart of the scalar Laplacian. In the reduced model, the state variable is the strain trace, and thermodynamic equilibrium is expressed by means of a direct relation between the interface mean curvature H and the crystal Gauss curvature R. In particular it shows polygonization effect as a result of the maximal dissipation postulate. In the tensor model, the equilibrium equation are also interpreted in the hardening limit, that is, as clustering takes place. Moreover the optimizsation problem adjoint state at stationnarity shows to correspond to the residual strain, i.e., the nonelastic strain which as added to strain model variable renders the total strain compatible.

If times permits, several physical interpretation will be proposed. This work has been/will be published in [5, 6].

References

[1] E. Kröner, Continuum theory of defects. Physiques des défauts, Les Houches session XXXV (Course 3), 1980.
[2] G. Allaire, F.Jouve, N. Van Goethem, Damage and fracture evolution in brittle materials by shape optimization methods. J. Comput. Physics **33**(16) (2011), 510–5044.
[3] R. Müller, Differential Harnack inequalities and the Ricci flow. Lecture Series in Mathematics, EMS, 2006.
[4] N. Van Goethem, The non-Riemannian dislocated crystal: a tribute to Ekkehart Kröner's (1919-2000). J. Geom. Mech. **3**(2) (2010), 303-320.
[5] N. Van Goethem, Shape optimization for a time-dependent 4th-order equation in a dislocation model. C. R. Acad. Sci. Paris, Ser. I 340 **349** (2011), 923–927.
[6] N. Van Goethem, Thermodynamic forces in single crystals with dislocations. Z. Ang. Math. Phys. (2013).

Convergence rates of order one perturbations in the 1-d Cahn Hilliard equation

Maria G. Westdickenberg

RWTH Aachen, Germany

We offer a new method for obtaining sharp relaxation rates for a gradient flow system using natural algebraic and differential relationships among the triple of distance, energy, and dissipation. We develop and apply the method in the context of the 1-d Cahn Hilliard equation on the line. This work is joint with Felix Otto.

Entropic flows, stochastic perturbations and microscopic models

Johannes Zimmer

University of Bath, UK

As starting point, different ways to link some PDEs of diffusive type to particle models will be reviewed. Typically, the PDE will be related to a minimisation problem of a large deviation rate function from probability. Here various approaches are possible and will be sketched.

The underlying microscopic process contains more information, notably fluctuations around the minimum state described by the deterministic PDE. Can stochastic terms be derived which model this additional information, in a way that is compatible to the limit passage via large deviations (and the geometric structure, such as the Wasserstein setting)? This question will be investigated for the simplest possible case of linear diffusion.

Becker-Steinberger, Katharina	Universität Ulm, Institut für Numerische Mathematik Ulm, Germany katharina.becker-steinberger@uni-ulm.de http://www.uni-ulm.de/mawi/mawi-numerik/mitarbeiter/dipl-math-katharina-becker-steinberger.html
Bonaschi, Giovanni	Technische Universiteit Eindhoven Centre for Analysis, Scientific Computing, and Applications Eindhoven, The Netherlands g.a.bonaschi@tue.nl
Braides, Andrea	University of Rome Department of Mathematics Rome, Italy braides@mat.uniroma2.it http://www.mat.uniroma2.it/~braides/
Brokate, Martin	Technische Universität München Zentrum Mathematik Munich, Germany brokate@ma.tum.de http://www-m6.ma.tum.de/Lehrstuhl/MartinBrokate
Bukal, Mario	University of Zagreb Faculty of Electrical Engineering and Computing, Department of Control and Computer Engineering Zagreb, Croatia mario.bukal@fer.hr http://www.fer.unizg.hr/mario.bukal
Carrillo, José Antonio	Imperial College Department of Mathematics London, UK carrillo@imperial.ac.uk http://www2.imperial.ac.uk/~jcarrill/
Davoli, Elisa	Carnegie Mellon University Department of Mathematical Sciences Pittsburgh, U.S.A. edavoli@andrew.cmu.edu
Disser, Karoline	Weierstrass Institute for Applied Analysis and Stochastics Partial Differential Equations Berlin, Germany karoline.disser@wias-berlin.de http://www.wias-berlin.de/~disser/

Participants

Duong, Hong
Technische Universiteit Eindhoven
Centre for Analysis, Scientific Computing, and Applications
Eindhoven, The Netherlands
m.h.duong@tue.nl

Feireisl, Eduard
Institute of Mathematics
Evolution Differential Equations
Prague, Czech Republic
feireisl@math.cas.cz
http://www.math.cas.cz/homepage/main_page.php?id_membre=37

Fiebach, André
Weierstrass Institute for Applied Analysis and Stochastics
Numerical Mathematics and Scientific Computing
Berlin, Germany
andre.fiebach@wias-berlin.de
http://www.wias-berlin.de/~fiebach/

Gelantalis, Michael
RWTH Aachen University
Lehrstuhl für Mathematik
Aachen, Germany
gelantalis@math1.rwth-aachen.de
http://www.math1.rwth-aachen.de/de/mitarbeiter

Heinz, Sebastian
Weierstrass Institute for Applied Analysis and Stochastics
Partial Differential Equations
Berlin, Germany
sebastian.heinz@wias-berlin.de
http://www.wias-berlin.de/~heinz/

Herrmann, Michael
Universität des Saarlandes
Fachrichtung Mathematik
Saarbrücken, Germany
michael.herrmann@math.uni-sb.de
http://www.math.uni-sb.de/ag/herrmann/

Jüngel, Ansgar
Vienna University of Technology
Institute for Analysis and Scientific Computing
Vienna, Austria
juengel@asc.tuwien.ac.at
http://www.asc.tuwien.ac.at/~juengel/

Klein, Olaf
Weierstrass Institute for Applied Analysis and Stochastics
Thermodynamic Modeling and Analysis of Phase Transitions
Berlin, Germany
olaf.klein@wias-berlin.de
http://www.wias-berlin.de/~klein/

Lengeler, Daniel
Universität Regensburg
Fakultät für Mathematik
Regensburg, Germany
daniel.lengeler@mathematik.uni-regensburg.de

Liero, Matthias
Weierstrass Institute for Applied Analysis and Stochastics
Partial Differential Equations
Berlin, Germany
matthias.liero@wias-berlin.de
http://www.wias-berlin.de/people/liero/

Markowich, Peter A.
University of Cambridge
Department of Applied Mathematics and Theoretical Physics (DAMTP)
Cambridge, UK
P.A.Markowich@damtp.cam.ac.uk
http://www.damtp.cam.ac.uk/people/p.markowich/
and
University of Vienna
Faculty of Mathematics
Vienna, Austria

Matthes, Daniel
Technische Universität München
Zentrum Mathematik
Munich, Germany
matthes@ma.tum.de
http://www-m8.ma.tum.de/personen/matthes/

Meyries, Martin
Universität Halle
Junior Research Group “Partial Differential Equations and Applications”
Halle, Germany
martin.meyries@mathematik.uni-halle.de
pde.mathematik.uni-halle.de

Mielke, Alexander
Weierstrass Institute for Applied Analysis and Stochastics
Partial Differential Equations
Berlin, Germany
alexander.mielke@wias-berlin.de
http://www.wias-berlin.de/people/mielke/

Mittnenzweig, Markus
Freie Universität Berlin
Berlin, Germany
markus.mittnenzweig@gmx.de

Participants

Mora, Maria Giovanna
University of Pavia
Department of Mathematics
Pavia, Italy
mariagiovanna.mora@unipv.it
http://www-dimat.unipv.it/mora/

Neukamm, Stefan
Weierstrass Institute for Applied Analysis and Stochastics
Partial Differential Equations
Berlin, Germany
stefan.neukamm@wias-berlin.de
http://www.wias-berlin.de/people/neukamm/

Platzek, Florian
Weierstrass Institute for Applied Analysis and Stochastics
Partial Differential Equations
Berlin, Germany
florian.platzek@wias-berlin.de
http://www.wias-berlin.de/~platzek/

Renger, Michiel
Weierstrass Institute for Applied Analysis and Stochastics
Partial Differential Equations
Berlin, Germany
michiel.renger@wias-berlin.de
http://www.wias-berlin.de/~renger/

Rocca, Elisabetta
Weierstrass Institute for Applied Analysis and Stochastics
ERC Group "Entropy Formulation of Evolutionary Phase Transitions"
Berlin, Germany
elisabetta.rocca@wias-berlin.de
http://www.wias-berlin.de/~rocca/

Rossi, Riccarda
University of Brescia
Department of Mathematics
Brescia, Italy
riccarda.rossi@unipv.it
http://dm.ing.unibs.it/~riccarda.rossi/

Savaré, Giuseppe
University of Pavia
Department of Mathematics
Pavia, Italy
giuseppe.savare@unipv.it
http://www.imati.cnr.it/savare/

Schlichting, André
University of Bonn
Institute for Applied Mathematics
Bonn, Germany
schlichting@iam.uni-bonn.de
http://www.andre-schlichting.de/

Sharma, Upanshu
Technische Universiteit Eindhoven
Centre for Analysis, Scientific Computing, and Applications
Eindhoven, The Netherlands
u.sharma@tue.nl

Stefanelli, Ulisse
University of Vienna
Faculty of Mathematics
Vienna, Austria
ulisse.stefanelli@univie.ac.at

Stephan, Holger
Weierstrass Institute for Applied Analysis and Stochastics
Numerical Mathematics and Scientific Computing
Berlin, Germany
holger.stephan@wias-berlin.de
http://www.wias-berlin.de/people/stephan/

Theil, Florian
University of Warwick
Mathematics Institute
Warwick, UK
F.Theil@warwick.ac.uk
http://www2.warwick.ac.uk/fac/sci/maths/people/staff/florian_theil/

Toader, Rodica
University of Udine
Department of Mathematics and Informatics
Udine, Italy
rodica.toader@uniud.it
http://people.uniud.it/page/rodica.toader

Van Goethem, Nicolas
International School of Advanced Studies (SISSA)
Mathematical Analysis, Modeling and Applications
Trieste, Italy
vangoeth@sissa.it
http://people.sissa.it/~vangoeth/

Westdickenberg, Maria G.
Aachen University (RWTH)
Department of Mathematics
Aachen, Germany
maria@math1.rwth-aachen.de
http://www.math1.rwth-aachen.de/~westd/

Zimmer, Johannes
University of Bath
Mathematics Institute
Bath, UK
zimmer@maths.bath.ac.uk
http://www.maths.bath.ac.uk/~jz203/

For Your Notes

For Your Notes