Griffith's fracture criterion in piezoelectric ceramics

Anna-Margarete Sändig, Stefanie Braun, Winfried Geis

Institute of Applied Analysis and Numerical Simulation
Universität Stuttgart
D- 70569 Stuttgart, Germany
saendig@ians.uni-stuttgart.de

We discuss whether Griffith's fracture criterion for elastic bodies is applicable to piezoelectric materials where mechanical and electric fields interact. The key point is to consider an appropriate total potential energy which reflects fracture to be a mechanical process and regards the influence of electric and mechanical loads. Furthermore, the existence of an energy minimum in different configurations has to be assured in order to characterise when a crack is stationary. This last property admits reformulating the Griffith's criterion in terms of the energy release rate, which can be expressed as a volume integral (Griffith's formula) or a path integral (J-integral). Starting from the well known linear Voigt model we analyse whether the enthalpy functional (electric Gibbs energy), which is often used as the total energy functional, satisfies the above conditions. The answer is negative, i.e., due to the nonconvexity of the enthalpy density the existence of a minimiser cannot be assured. Therefore, we suggest to take the positive definite part of the enthalpy functional. This leads to an additional constitutive law, namely, a linear relation between strain and electric field, which means that higher order piezo feedback is neglected [1]. The resulting energy functional is a modified Helmholtz free energy. We formulate corresponding modified Euler-Lagrange equations [2] and present numerical results.

References

[1] S. Mahlberg *Griffith'sches Bruchkriterium in Piezokeramiken.* Diplomarbeit, IANS, University Stuttgart 2007.

[2] S. Braun and A.-M. Sändig *Griffith' fracture criterion in piezoelectric ceramics*, Bericht 2007/015 des Instituts für Angewandte Analysis und Numerische Simulation der Universität Stuttgart, 2007