A GRADIENT FLOW APPROACH TO CAHN-HILLIARD AND LUBRICATION EQUATIONS

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Scalar degenerate fourth order parabolic equations of the type

$$\partial_t \rho = -\operatorname{div}\big(\mathbf{m}(\rho)\nabla\Delta\rho\big) + \Delta g(\rho)$$

for a density function $\rho(t,x)$ on a domain Ω can *formally* be considered as gradient flow of a free energy functional with respect to an appropriate scalar product. Under certain assumptions on the non-negative mobility function $\mathbf{m}$, this idea can be made rigorous: there exists a genuine metric on the space of probability densities that induces the desired scalar product, which then allows to prove the existence of weak solutions of the PDE as limits of curves of steepest decent of the free energy, and to study their equilibration behavior by means of the calculus of variations. In this talk, we discuss two such PDEs: the first is a thin film model (corresponding to the classical Wasserstein distance), and the second is a Cahn-Hilliard equation (corresponding to generalized distances defined by Savaré et al).

This is joint work with Giuseppe Savaré and Stefano Lisini from Pavia (IT), and Robert McCann from Toronto (CA).