

Analysis and approximation of incompressible chemically reacting non-Newtonian fluids

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We consider a system of nonlinear partial differential equations modelling the motion of an incompressible chemically reacting non-Newtonian fluid. The governing system consists of a convection-diffusion equation for the concentration and the generalized Navier–Stokes equations, where the viscosity coefficient is a power-law type function of the shear-rate, and the power-law index depends on the concentration. This system of nonlinear partial differential equations arises in mathematical models of the synovial fluid found in the cavities of movable joints. We first discuss the existence of weak solutions of the model. Then we construct a finite element approximation of the model and perform the mathematical analysis of the numerical method. Key technical tools include discrete counterparts of the Bogovskii operator, De Giorgi type regularity theorem, and the Acerbi–Fusco Lipschitz truncation of Sobolev functions, in function spaces with variable-integrability exponents.