Spectral asymptotics for mixed problems and for crack problems on infinite cylinders

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We start by computing the spectral asymptotics for the ground state eigenvalue of the two-dimensional problem

$$\begin{aligned} -\Delta u &= \lambda u \qquad \text{on } \Omega := \mathbb{R} \times (0,1) \\ \partial_n u &= 0, \qquad \text{on } \Gamma_N := (-\ell, \ell) \times \{1\} \\ u &= 0, \qquad \text{on } \Gamma_D := \partial\Omega \backslash \Gamma_N, \end{aligned}$$

which describes the behaviour of two quantum wave guides coupled through a small Neumann window of size ℓ. By a matching procedure of two different asymptotic expansion it was proved by I. Yu. Popov ('99) that the ground state satisfies

$$\sqrt{\pi^2 - \lambda(\ell)} = \ell^2 \cdot \frac{\pi^3}{2} + \mathcal{O}(\ell^4 \ln \ell) \qquad \text{as } \ell \to 0. \tag{1}$$

We give a new proof of the formula based on an expansion of the corresponding Dirichlet-to-Neumann operator and a subsequent Birman-Schwinger analysis. Furthermore, we want to generalise the approach to elastic crack problems.