SCALAR CONSERVATION LAWS ON CONSTANT AND TIME-DEPENDENT RIEMANNIAN MANIFOLDS

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Many physical systems from continuum mechanics can be modelled by nonlinear conservation laws. Typically, these partial differential equations of hyperbolic type are posed in Euclidean space. For some applications, though, the suitable domains turn out to be hypersurfaces or, more generally, Riemannian manifolds which, additionally, may change in time. Consider for example the shallow water equations on the sphere as a model for the global air and water flow, the flow of oil on a moving water surface or surfactants on the interfacial hypersurface between two phases in multiphase flow. Scalar conservation laws have been established as a good model problem for studying the nonlinear effects in such systems. We establish well-posedness for scalar conservation laws on closed manifolds M endowed with a constant or a time-dependent Riemannian metric for initial values in $L^\infty(M)$. Furthermore, we derive estimates of the total variation of the solution for initial values in BV, and we give, in the case of a time-independent metric, a simple geometric characterisation of flux functions that give rise to total variation diminishing estimates.