Weierstrass Institute for Applied Analysis and Stochastics

Karsten Tabelow[(1)], Henning U. Voss[(2)], Jörg Polzehl[(1)]

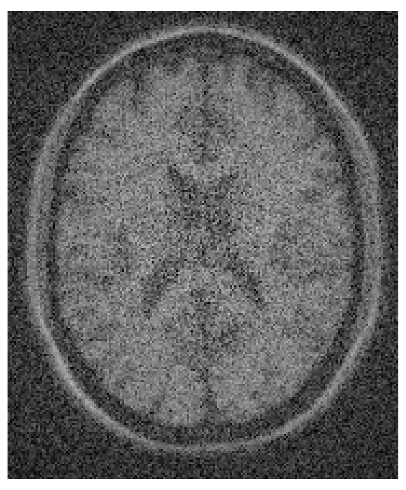
Local estimation of noise standard deviation in MRI images using propagation separation

HBM 2014 - Poster #1634-MoTue

Objective of this research

We present a method for local estimation of the signal-dependent noise level in magnetic resonance images. The procedure uses a multi-scale approach to adaptively infer on local neighborhoods with similar data distribution. It exploits a maximum-likelihood estimator for the local noise level. We evaluated the validity of the method on repeated diffusion data of a phantom (not shown here) and simulated data. The method is especially useful for **low SNR** and **high MR image resolution**. For example, in diffusion MRI it can be used for **improved diffusion tensor (DTI) estimates** or for **improved noise reduction**.

We demonstrate the improvements by examples.



- Noise variation can be estimated from image background, cf. Aja-Fernandez (2009), MRI 27, 1397-1409.
- However, noise variation is not homogeneous over image!
- Missing background in restricted field-of-view techniques!
- Local noise variation determines a) local image quality, b) bias due to skewed signal distribution, c) effectiveness of noise reduction methods.

Data acquisition

- Complex Gaussian noise in k-space for each coil
- Signal distribution S/σ is (approximately) non-central $\chi_{2L,\eta}$ with density p_S for S:

$$\frac{S^L \eta^{(1-L)}}{\sigma^{(L+1)}} e^{-\frac{1}{2}\left(\frac{S^2}{\sigma^2}+\eta^2\right)} I_{L-1}\left(\frac{\eta S}{\sigma}\right)$$

Special cases

a) Single coil: $L=1$, global σ (Rician)

b) Sum-of-squares: local L and σ (correlation)

c) GRAPPA: local L and σ

d) SENSE1: $L=1$, local σ

Maximum Likelihood estimator

- Given signal values S_j in voxel j: $(S_j/\sigma_j) \sim \chi_{2L,\eta_j}$
- Weighted log-likelihood $(\hat{\sigma}_i, \hat{\eta}_i) =$

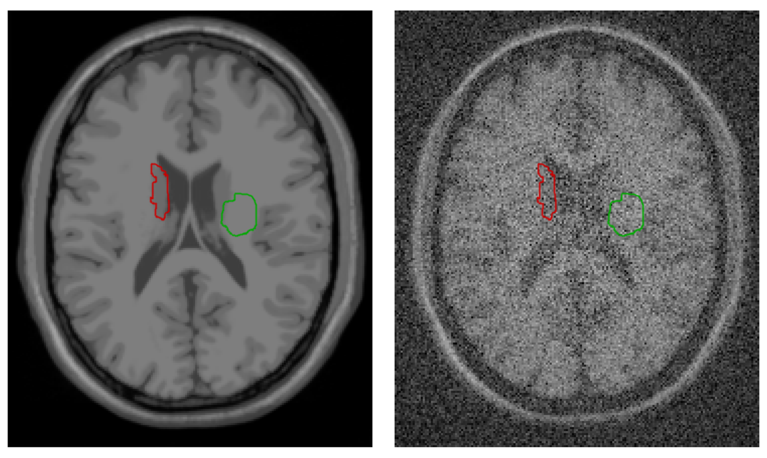
$$\underset{(\eta,\sigma)}{\operatorname{argmax}} \sum_j w_{ij} \log p_S(S_j; \eta, \sigma, L)$$

cf. Sijbers (1998) MRI, 16, 87-90.

- $w_{ij} = 1$, if $\eta_i = \eta_j$, 0 otherwise
- Assume: local constant (homogeneous) η
- Assume: slowly varying σ
- Assume: known L

Homogeneity region definition

- How to define region of homogeneous η?



- ... in noisy data?
- Homogeneity region defined by weights w_{ij}

Propagation separation

- Sequential multi-scale algorithm
- Compute adaptive weights

$$w_{ij}^{(k)} = K_{\text{loc}}\left(\frac{\|i-j\|}{h^{(k)}}\right) K_{\text{st}}\left(\frac{s_{ij}^{(k-1)}}{\lambda}\right)$$

where $s_{ij}^{(k-1)}$ evaluates the statistical difference between the estimates in voxel i and j from step $k-1$.

- Estimate $\hat{\sigma}_i, \hat{\eta}_i$ by weighted log-likelihood.
- Iterate over increasing scales $\{h^{(k)}\}_{k=0}^{k^*}$

Evaluation of new method using simulated MR data (T1)

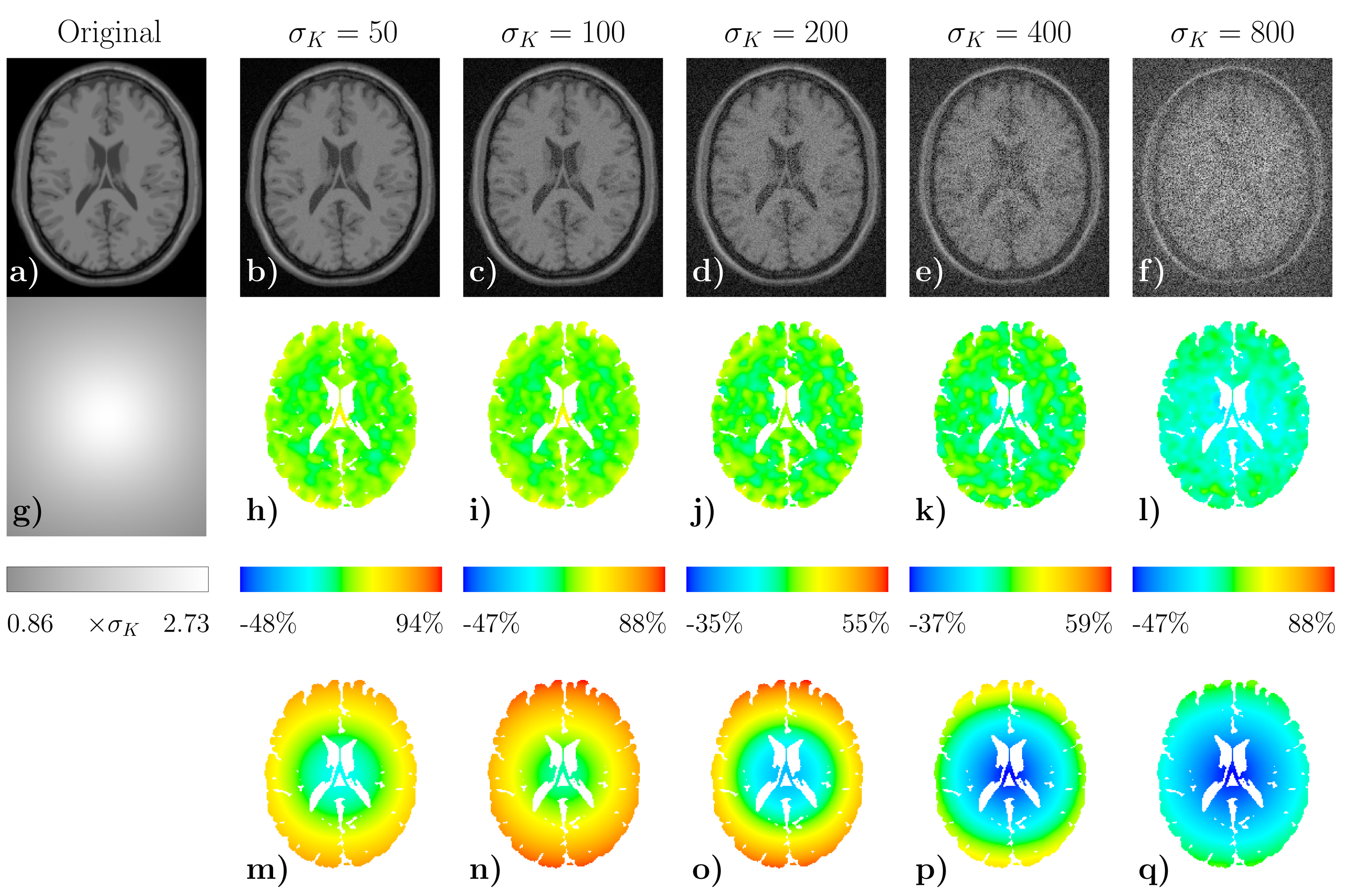


Figure: Simulation results for a slice of the BrainWeb MR volume. a) Original slice. b)-f) slice after adding complex Gaussian noise with standard deviation σ_K in k-space and SENSE1 reconstruction from 8 receiver coils (positive correlation between receiver coils). g) image of (locally varying) effective σ h)-l) relative error of local estimates $\hat{\sigma}_i$ using the proposed method. m)-q) relative error of local estimates $\hat{\sigma}_i$ using the method described in Aja-Fernandez (2013), MRI 31, 272-285. The errors are given on a log-scale.

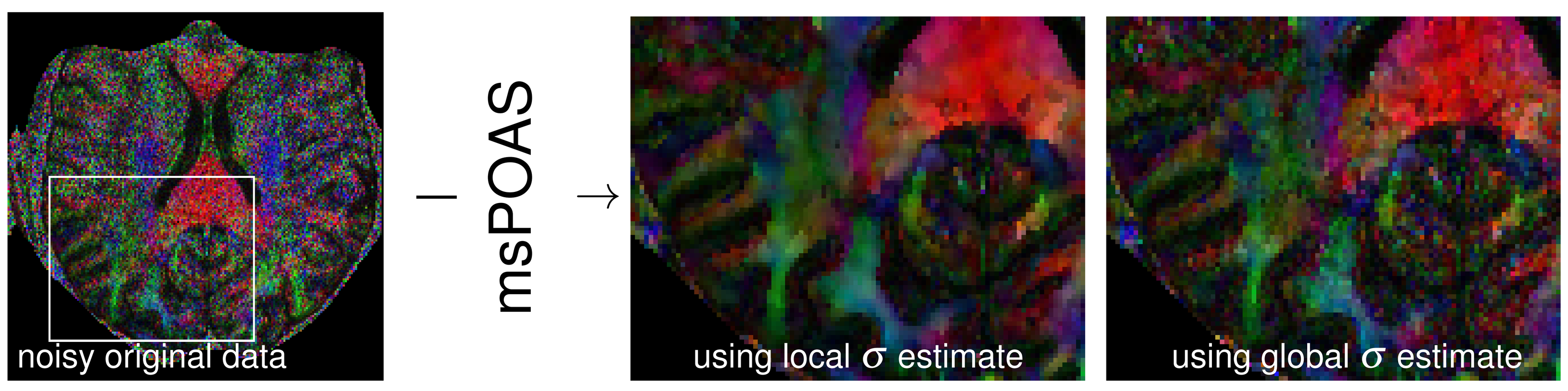
Conclusions:

- New method enables *local* estimation of scale parameter in the non-central χ distribution in MR imaging, good agreement with true values in simulation.
- Good agreement with estimate from repeated measurements, see Tabelow et al. (2014).
- Estimation inside white and gray matter regions.

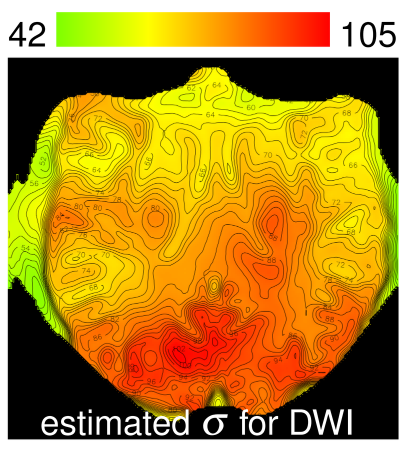
MRI acquisition (diffusion weighted dataset)

We re-analyzed a dataset described already in Becker (2012) and Becker(2013). Data were acquired from a whole body 7T MAGNETOM scanner (Siemens Healthcare) with a maximum gradient amplitude of 70mT/m and a maximum slew rate of 200T/m/s (SC72, Siemens Healthcare, Erlangen, Germany). The scan was performed using a single channel transmit, 24-channel receive phased array head coil (Nova Medical, Wilmington, MA, USA). An optimized monopolar Stejskal-Tanner sequence according to Morelli (2010) together with the ZOOPPA approach described in Heidemann (2012) was used with TR 14.1s, TE 65ms, BW 1132Hz/pixel, and ZOOPPA acceleration factor of 4.6. A total of 91 slices with 10% overlap were acquired at a field-of-view (FoV) of $143 \times 147\text{mm}^2$ resulting in an isotropic high resolution of 800μm. Diffusion weighting gradients were applied along 60 different directions at a b-value of 1000s/mm^2. 7 interspersed non-diffusion weighted images were acquired. The scan was repeated 4 times. The subject was a healthy adult volunteer after obtaining written informed consent in accordance with the ethical approval from the University of Leipzig. Total acquisition time was 65min.

Application I - Local variance reduction by msPOAS, see Poster #1635-MoTue



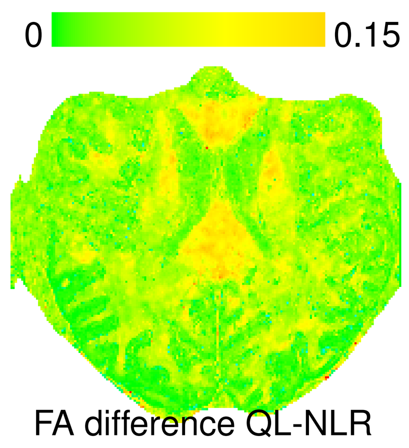
Application II - DTI by quasi-likelihood (QL) instead of non-linear regression (NLR)



DTI model: $\zeta_{p,i} = \zeta_i^0 \exp(-b_p \vec{g}_p^\top \mathscr{D}_i \vec{g}_p)$

$$NLR: \quad \operatorname{argmin} \sum_p (S_{i,p} - \zeta_{i,p})^2$$

$$QL: \quad \operatorname{argmin} \sum_p \left[\frac{\left(S_{p,i} - \mu\left(\frac{\zeta_{p,i}}{\sigma_{p,i}}, \sigma_{p,i}, L_i\right)\right)^2}{v\left(\frac{\zeta_{p,i}}{\sigma_{p,i}}, \sigma_{p,i}, L_i\right)} \right]$$



Software

- Free download: `http://www.nitrc.org/projects/rdti/`
- ... and `http://cran.r-project.org/web/packages/dti/index.html`
- J. Polzehl, K. Tabelow (2011), 'Beyond the Gaussian Model in Diffusion-Weighted Imaging: The package dti', *J. Statist. Software*, vol. 44, issue 12. (Explaining the usage of the package for HARDI)
- J. Polzehl, K. Tabelow (2009), 'Structural adaptive smoothing in diffusion tensor imaging: The R package dti', *J. Statist. Software*, vol. 31, pp. 1–24. (Explaining the usage of the package for DTI)

Further reading

- K. Tabelow, H.U. Voss, J. Polzehl, (2014) Local estimation of the noise level in MRI using structural adaptation, WIAS Preprint No. 1947.
- S. Becker, K. Tabelow, S. Mohammadi, N. Weiskopf, J. Polzehl, (2014) Adaptive smoothing of multi-shell diffusion- weighted magnetic resonance data by msPOAS, NeuroImage vol. 95, pp. 90–105.
- S. Becker, K. Tabelow, H.U. Voss, A. Anwander, R.M. Heidemann, J. Polzehl, (2012) Position-orientation adaptive smoothing of diffusion weighted magnetic resonance data (POAS), *Med. Image Anal.* 16, pp. 1142–1155.

[1] Weierstrass Institute · karsten.tabelow@wias-berlin.de [2] Weill Cornell Medical College, New York, NY