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ON ASYMPTOTIC MINIMAXITY OF KOLMOGOROV AND OMEGA SQUARE TESTS

M.Ermakov

Summary. We consider the problem of hypothesis testing about a value of functional. For a given functional T the problem is to test a hypothesis $T(P) = 0$ versus alternatives $T(P) > b_0 > 0$ where P is an arbitrary probability measure. Under the natural assumptions we show that the test statistics $T(\hat{P}_n)$ depending on the empirical probability measures $\hat{P}_n$ are asymptotically minimax. Since the sets of alternatives is fixed the asymptotic minimaxity is considered in the senses of Bahadur and Hodges–Lehmann efficiencies. In particular the functional T can be the functional corresponded to the test statistics of Kolmogorov and omega-square tests.

Key words and phrases: large deviations, nonparametric hypothesis testing, asymptotically minimax hypothesis testing, Bahadur efficiency, Hodges–Lehmann efficiency, Kolmogorov test, omega-square test.

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1.Introduction. In nonparametric hypothesis testing a statistician has not usually the information about the parametric structure of sets of alternatives. From this point it is natural to study the properties of nonparametric tests in essentially nonparametric setting. A natural approach to the assignment of nonparametric sets of alternatives has been proposed by Ch.Stein (1956). For the setting under consideration this approach is as follows.

Let S be a separable Hausdorff space, B the σ–field of Borel sets in S, Λ the set of all probability measures (pms) on (S,B) and T a real function on Λ. Let $X_1,\dots,X_n$ be i.i.d.r.v.'s with pm $P\in\Lambda$ and let $\hat{P}_n$ be their empirical pm. Suppose the problem is to test a hypothesis $P\in\Omega(0)=\Omega_T(0)=\{P:T(P)=0,P\in\Lambda\}$ versus alternatives $P\in\Phi(b_0)=\Phi_T(b_0)=\{P:T(P)>b_0>0,P\in\Lambda\}$. For this problem we show that the sequence of test statistics $T(\hat{P}_n)$ are asymptotically minimax in the senses of Bahadur and Hodges–Lehmann efficiencies. In the case of Kolmogorov and omega–square tests the functional T equals respectively.

$$T(P)=\max\{|F(x)-F_0(x)|q(F_0(x))\,:\,x\in(0,1)\}$$

and

$$T(P)=\int_0^1(F(x)-F_0(x))^2q(F_0(x))dF_0(x).$$

Here F and F_0 are the distribution functions (dfs) of pms P and P_0 respectively, and q is a nonnegative continuous function on $[0,1]$.

This setting is the setting of hypothesis testing about a value of functional. This problem has not obtained such a comprehensive development as in estimation (see Levit (1974), Koshevnick and Levit (1976), Millar (1983), Ibragimov and Khasminskii (1991), van der Vaart (1991), Bickel, Klaassen, Ritov and Wellner (1993), and others). We can mention only the papers of Ermakov (1990),(1992),(1993). In this papers the lower bound for the Pitman efficiency has been indicated and results about asymptotic minimaxity of Kolmogorov, omega-square and chi-square tests has been announced.

The study of Bahadur and Hodges–Lehmann efficiencies of test statistics is a traditional theme in the theory of hypothesis testing. We should mention the papers of Bahadur (1971), Hodges–Lehmann (1956), Abrahamson (1967) , Mogulskii (1977), Nikitin (1979), Groeneboom and Shorack (1981), Kallenberg and Ledwina (1987), and others.

2.Main Results.. For any pms $P,Q\in\Lambda$ define the Kullback–Leibler information

$$K(Q,P)=\int_S q\log q dP \qquad \text{if} \qquad Q<<P \qquad (2.1)$$

and $K(Q,P)=\infty$ otherwise. Here $q=dQ/dP$. For any $P\in\Lambda$ and any $\Omega\subset\Lambda$, $\Phi\subset\Lambda$ denote $K(\Omega,P)=\inf\{K(Q,P):Q\in\Lambda\}$ and $K(\Omega,\Phi)=\inf\{K(\Omega,P):P\in\Phi\}$.

Introduce on Λ the topologies τ and τ_c of weak convergence. The topology τ is used traditionally in the theory of large deviations (see Groeneboom, Oosterhoff and Ruymgaart 1979). The topology τ_c will be necessary for the receipt of the more subtle results which could not be obtained on the base of the topology τ. We say that a sequence of pms P_n converges to a pm P in the topology τ (the topology τ_c) iff for each bounded measurable function f (continuous bounded function f respectively)

$$\lim_{n\to\infty} \int_S f dP_n = \int_S f dP. \qquad (2.2)$$

If otherwise is not stipulated all topological properties will be considered with respect to topology τ. The closure and the interior of a set Ω in the topology τ will be denoted by cl (Ω) and int (Ω) respectively.

The results will be given for the k-sample setting since their generalization on this case is trivial. The changes in the setting and notations are as follows. Let $X_{i1}, \ldots, X_{in_i}$ be i.i.d.r.v.'s taking values in S according to pm $P_i \in \Lambda$, $1 \le i \le k$. Denote $n = n_1 + \ldots + n_k$ and suppose that $n_i/n \to \nu_i > 0$ as $n \to \infty$ for all $1 \le i \le k$. Put $\hat{P}_n = \hat{P}_{1n_1} \times \ldots \times \hat{P}_{kn_k}$ where $\hat{P}_{in_i}$ is the empirical pm of $X_{i1}, \ldots, X_{in_i}$, $1 \le i \le k$. Λ is endowed with the topology τ and Λ^k is given the product topology. For a given functional $T : \Lambda^k \to R^1$ and any $a, b \in R^1$ define the sets $\Omega(a) = \{P : T(P) < a,\ P \in \Lambda^k\}$ and $\Phi(b) = \{P : T(P) \ge b,\ P \in \Lambda^k\}$. As mentioned the problem is to test a hypothesis $P \in \Omega(0)$ versus alternatives $P \in \Phi(b_0)$, $b_0 > 0$.

For any $Q = Q_1 \times \ldots \times Q_k$, $P = P_1 \times \ldots \times P_k \in \Lambda^k$ denote

$$I_\nu(Q, P) = \sum_{i=1}^{k} \nu_i K(Q_i, P_i). \qquad (2.3)$$

For any $P \in \Lambda^k$ and any $\Omega \subset \Lambda^k$, $\Phi \subset \Lambda^k$ put $I_\nu(\Omega, P) = \inf\{I_\nu(Q, P) : Q \in \Omega\}$, $I_\nu(\Omega, \Phi) = \inf\{I_\nu(\Omega, P) : P \in \Phi\}$. We show that the asymptotically minimax lower bounds can be given in the terms of functional $I_\nu(\Omega, \Phi)$.

For any test W_n denote $\alpha(W_n)$ its type I error probability and $\beta(W_n, Q)$ its type II error probability under the alternative $Q \in \Phi(b)$. Put $\beta(W_n, b) = \sup\{\beta(W_n, Q) : Q \in \Phi(b)\}$. If L_n is the test statistics of test W_n then denote $\beta(\alpha, L_n, Q) = \beta(W_n, Q)$, $\beta(\alpha, L_n, b_0) = \beta(W_n, b_0)$ where $\alpha = \alpha(W_n)$.

We say that a sequence of test statistics $L = \{L_n\}$ have the uniform Hodges–Lehmann index $d(L, b_0)$ if

$$d(L, b_0) = -2 \lim_{n\to\infty} n^{-1} \log \beta(\alpha, L_n, b_0). \qquad (2.4)$$

The uniform Bahadur slope $e(L, b_0)$ is defined similarly

$$e(L, b_0) = -2 \lim_{n\to\infty} n^{-1} \log r(\beta_1, L_n, b_0) \qquad (2.5)$$

where $r(\beta_1, L, b_0) = \sup\{\alpha : \beta(\alpha, L, b_0) < \beta_1\}$, $\beta_1 \in (0, 1)$.

Note that in these definitions we assume that the left handsides of (2.4),(2.5) do not depend on α and β_1 respectively. This assumption is usually satisfied for the test statistics.

We say that the sequence of test statistics $L = \{L_n\}$ is asymptotically minimax in the sense of Hodges–Lehmann efficiency (Bahadur efficiency respectively) if for any sequence of test statistics $V = \{V_n\}$ it holds

$$\limsup_{n\to\infty} \log \beta(\alpha, V_n, b_0)/\log \beta(\alpha, L_n, b_0) \geq 1$$

for all $\alpha \in (0,1)$ (respectively

$$\limsup_{n\to\infty} \log r(\beta, V_n, b_0)/\log r(\beta, L_n, b_0) \geq 1$$

for all $\beta \in (0,1)$).

It is clear, that $d(V, b_0) \leq d(L, b_0)$ and $e(V, b_0) \leq e(L, b_0)$.

Theorem 2.1. *Let $T : \Lambda^k \to R^1$ be a nonnegative continuous functional and let one of the following properties be satisfied*

i). $\Omega(0) = \{P_0\}$, $P_0 \in \Lambda^k$.

ii). S is a compact set. There exists b_1, $b_1 < b_0$, such that the sets $\Phi(b)$, $b_1 < b < b_0$, and $\Omega(0)$ are closed in the topology τ_c.

Then the sequence of test statistics $T(\hat{P}_n)$ is asymptotically minimax in the sense of Bahadur efficiency and $e(T, b_0) = I_\nu(\Phi(b_0), \Omega(0))$.

Theorem 2.2. *Let $T : \Lambda^k \to R^1$ be a nonnegative continuous functional. Let S be a compact set and let there exist $a_0 > 0$ such that the sets $\Phi(b_0)$, $\Omega(a)$ with $0 \leq a \leq a_0$ are closed in the topology τ_c. Then the sequence of test statistics $T(\hat{P}_n)$ are asymptotically minimax in the sense of Hodges-Lehman efficiency and $d(T, b_0) = I_\nu(\Omega(0), \Phi(b_0))$.*

Theorem 2.1 are easily transfered on the problems of testing complex nonparametric hypothesis. As an example consider the problem of testing a hypothesis of homogenity with $k = 2$ and $S = [0,1]$.

Denote Ψ the set of all dfs and Ψ_c the set of all continuous dfs. On a functional $T : \Lambda^2 \to R^1$ define the functional $T_c : \Psi^2 \to R^1$ such that $T_c(H_1, H_2) = T(Q_1, Q_2)$ for any dfs H_1,H_2 having pms Q_1,$Q_2 \in \Lambda$ respectively. Denote $H = \nu_1 H_1 + \nu_2 H_2$. For any $H \in \Psi_c$ define the inverse function H^{-1} such that $H^{-1}(t) = s$, $t = H(s)$ for any $t \in (0,1)$. Suppose that the functional T_c has the following properties which are assigned traditionally for the tests of homogenity .

A. For any H_1,$H_2 \in \Psi_c$ $T_c(H_1, H_2) = 0$ implies $H_1 = H_2$

B. For any H_1,$H_2 \in \Psi_c$ $T_c(H_1, H_2) = T_c(G_1, G_2)$ where $G_1(t) = H_1(H^{-1}(t))$, $G_2(t) = H_2(H^{-1}(t))$ for all $t \in (0,1)$.

Theorem 2.3. *Let $T : \Lambda^2 \to R^1$ be a continuous functional satisfying* A,B. *Then the sequence of test statistics $T(\hat{P}_n)$ is asymptotically minimax in the sense of Bahadur efficiency and $e(T, b_0) = I_\nu(\Phi(b_0), \Omega(0))$.*

The equivariance property B together with the assumption A allows us to modify easily the proof of Theorem 2.1 on this case. Thus the proof of Theorem 2.3 will be omitted.

3.Proofs of Theorems 2.1,2.2. Since the arguments for one and several samples are similar we consider only the one sample case, that is, $k = 1$.

Denote $\Pi = \Pi_m = \{S_j\}_1^m$ a finite partition of S consisting of Borel sets S_j, $1 \le j \le m$, and say that a partition Π_m embeded in a partition Π_l iff for each $S \in \Pi_m$ there exists $S' \in \Pi_l$ such that $S \subset S'$. If for each m a partition Π_{m+1} embedded in Π_m then we shall say that we have a sequence of embedded partitions Π_m.

For any $P, Q \in \Lambda$ denote

$$K(Q, P|\Pi) = \sum_{j=1}^{m} Q(S_j) \log Q(S_j)/P(S_j). \qquad (3.1)$$

For any set $\Omega \subset \Lambda$ and $P \in \Lambda$ put $K(\Omega, P|\Pi) = \inf\{K(Q, P|\Pi) \,|\, Q \in \Omega\}$. Note that $K(Q, P|\Pi_l) \le K(Q, P|\Pi_m)$ if Π_m embedded in Π_l.

The next Lemma is a refinement of Lemma 3.1 in Groeneboom, Oosterhoff and Ruymgaart (1979).

Lemma 3.1. *For any open set $\Omega \in \Lambda$ and any pm $P \in \Lambda$*

$$P(\hat{P}_n \in \Omega) < \exp\{-nK(\Omega, P|\Pi_m) + n\omega_m(n)\} \qquad (3.2)$$

where $\omega_m(n) \to 0$ as $n \to \infty$ and $\omega_m(n)$ does not depend on a choice of pm P and a partition Π_m.

The proof of Lemma 3.1 is obtained by a simple revision of estimates in the proof of Lemma 3.1 in Groeneboom, Oosterhoff and Ruymgaart (1979) and is omitted.

The next Lemma 3.2 and Theorem 3.1 have also the auxilliary character. Their proofs will be given in sections 4 and 5 respectively.

Lemma 3.2. *Let the assumptions of Theorem 3.1 be satisfied. Then*

d). $I_\nu(\Phi(b), \Omega(0))$ is continuous from the left in $b = b_0$.

dd). For any $b < b_0$ it holds

$$\lim_{n\to\infty} \sup_{P \in \Phi(b_0)} P(T(\hat{P}_n < b) = 0. \qquad (3.3)$$

Theorem 3.1. *Let S be a compact set, let $T : \Lambda^k \to R^1$ be a continuous functional and let the sets $\Phi(b)$, $\Omega(0)$ be closed in the topology τ_c. Then*

$$\lim_{n\to\infty} \sup_{P \in \Omega(0)} \log P(\hat{P}_n \in \Phi(b)) = -I_\nu(\Phi(b), \Omega(0)) \qquad (3.4)$$

Proof of Theorem 2.1. Suppose that *ii)* is valid and prove the upper bound, that is, $e(b_0, T) \le I_\nu(\Phi(b_0), \Omega(0))$.

Let W_n be a sequence of tests with the test statistics $T(\hat{P}_n)$ and let $\beta(W_n) = \beta$, $0 < \beta < 1$. Let Γ_n be the critical region of W_n. Then, by (3.3),

$$\Gamma_n = \Phi(b_0 - \epsilon_n) \qquad (3.5)$$

where $\epsilon_n \to 0$ as $n \to \infty$.

Fix ϵ, $0 < \epsilon < b_0$. By (3.4) we have

$$\lim_{n\to\infty} \sup_{P\in\Omega(0)} \log P(T(\hat{P}_n) > b_0 - \epsilon) = -I_\nu(\Phi(b_0 - \epsilon), \Omega(0)), \qquad (3.6)$$

By Lemma 3.2

$$\lim_{\epsilon\to 0} I_\nu(\Phi(b_0 - \epsilon), \Omega(0)) = I_\nu(\Phi(b_0), \Omega(0)) \qquad (3.7)$$

that completes the proof of upper bound.

If the assumption *i)* fulfilled then (3.6),(3.7) follows respectively from Theorem 3.1 and Lemma 3.3 in Groeneboom, Oosterhoff and Ruymgaart (1979).

Prove the lower bound. Fix $\epsilon \geq 0$. Then there exist pms $Q_0 \in \Phi(b_0)$, $P_0 \in \Omega(0)$ such that $I_\nu(Q_0, P_0) \leq I_\nu(\Phi(b_0), \Omega(0)) + \epsilon$. Consider the problem of testing the simple hypothesis $P = P_0$ versus the simple alternative $P = Q_0$ as the problem with "the least favourable" hypothesis and alternatives (see Lehmann (1986)). By Bahadur–Ragavachary inequality the Bahadur slope of likelihood ratio test in this problem equals $I_\nu(Q_0, P_0)$ that proves the lower bound.

Theorem 2.2 follows directly from Theorem 3.1. It suffices to note that if we interchange the sets of hypothesis and alternatives then the Bahadur efficiency becomes the Hodges–Lehmann one.

4.Proof of Lemma 3.2. In sections 4 and 5 $\Pi_m = \{S_{jm}\}_1^m$ denotes a sequence of embedded partitions such that the sets S_{jm}, $1 \leq j \leq m$, endows the σ–algebra B. Then, for any pms $P \in \Lambda$, $Q \in \Lambda$

$$K(Q, P) = \lim_{m\to\infty} K(Q, P|\Pi_m) \qquad (4.1)$$

Denote ∂A the boundary of the set A. Prove the following auxilliary Lemma.

Lemma 4.1. *Let S be a compact set. Let the sequences of pms P_l,$Q_l \in \Lambda$ converges in the topology τ_c to pms P,$Q \in \Lambda$ respectively. Then*

$$\liminf_{l\to\infty} K(Q_l, P_l) \geq K(Q, P) \qquad (4.2)$$

Proof. The arguments will be given under the assumptions that the pms P and Q has not atoms. This does not cause any principal differences in the proof.

Suppose that the partitions Π_m is choosed such that $P(\partial S_{jm}) = 0$, $Q(\partial S_{jm}) = 0$. Then $Q_l(S_{jm}) \to Q(S_{jm})$, $P_l(S_{jm}) \to P(S_{jm})$ as $l \to \infty$ for all $1 \leq j \leq m$, $1 \leq m \leq \infty$. Hence

$$\lim_{l\to\infty} K(Q_l, P_l|\Pi_m) = K(Q, P|\Pi_m) \qquad (4.3)$$

Now (4.1),(4.3) imply (4.2).

Prove (3.3). Suppose that (3.3) is not valid. Then, by (3.2), for each m there exist pms $P_m \in \Phi(b_0)$ and $Q_m \in \Omega(b)$ such that $Q_m(S_{jm}) = P_m(S_{jm})$ for all $1 \leq j \leq m$.

Denote A_m and B_m respectively the sets of all pms P_m and Q_m satisfying this property. For all $j \geq m$ define the sets $D_{mj} = \{v : v_{mj} = P(S_{jm}), v = \{v_{mj}\}_1^m, P \in A_j\}$ and put $M_m = \cap_{j=m}^{\infty} D_{mj}$. The sets M_m are closed and nonempty. Let the pm R be such that $\{R(S_{jm})\}_1^m \in M_m$ for all m and the sequences of pms $P_m \in A_m$, $Q_m \in B_m$ be such that $P_m(S_{jm}) = Q_m(S_{jm}) = R(S_{jm})$ for all $1 \leq j \leq m$. Then the sequences of pms P_m, Q_m converge to pm R. This implies $R \in \text{cl}(\Phi(b_0)) = \Phi(b_0)$ and $R \in \text{cl}(\Omega(b)) = \Omega(b)$. Thus we obtain the contradiction with the continuity of the functional T.

Prove d). If the condition i) of Theorem 2.1 is satisfied then d) follows from Lemma 3.3 in Groeneboom, Oosterhoff and Ruymgaart (1979). Suppose that the condition ii) is not valid. Then ,for any $\epsilon \geq 0$, there exist a sequence $b_l \leq b_0$ with $b_l \to b_0$ as $l \to \infty$ and sequences $P_l \in \Omega(0)$, $Q_l \in \Phi(b_l)$ such that

$$K(Q_l, P_l) \leq K(\Phi(b_0), \Omega(0)) - \epsilon. \qquad (4.4)$$

Since $\Omega(0)$, $\Phi(b)$ are the compact sets in the topology τ_c there exists subsequences P_{l_i}, Q_{l_i} converging in this topology to some pm $P_0 \in \Omega(0)$ and $Q_0 \in \Phi(b_0)$ respectively. By Lemma 4.1 we have

$$\liminf_{i \to \infty} K(Q_{l_i}, P_{l_i}) \geq K(Q_0, P_0)$$

that contradicts (4.4).

5.Proof of Theorem 3.1. The arguments are based on the same ideas. By Lemma 3.1

$$\sup_{P \in \Omega} P(\hat{P}_n \in \Phi(b)) \leq \exp\{-n(J_m + \omega_m(n))\} \qquad (5.1)$$

where $J_m = \inf\{K(\Phi(b), P|\Pi_m) \,|\, P \in \Omega(0)\}$.

Let the sequences pms $P_m \in \Omega(0)$, $Q_m \in \Phi(b)$ satisfy

$$\lim_{m \to \infty} K(Q_m, P_m|\Pi_m)/J_m = 1 \qquad (5.2)$$

and let the pms P_0, Q_0 be the limits points (in the topology τ_c) of the sequences P_m, Q_m respectively. Then, by Lemma 4.1 and (4.1),

$$\limsup_{m \to \infty} K(Q_0, P_0|\Pi_m)/J_m \leq 1 \qquad (5.3)$$

At the same time $P_0 \in \Omega(0)$, $Q_0 \in \Phi(b)$ and

$$K(Q_0, P_0) > K(\Phi(b), \Omega(0)) \geq J_m \qquad (5.4)$$

Now (4.1),(5.1)-(5.4) together implies (3.4).

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