Department for Mathematics and Computer Science
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Numerical Mathematics III – Partial Differential Equations

Exercise Problems 05

Attention: The approach for getting a solution has to be clearly presented. All statements have to be proved, auxiliary calculations have to be written down. Statements given in the lectures can be used without proof.

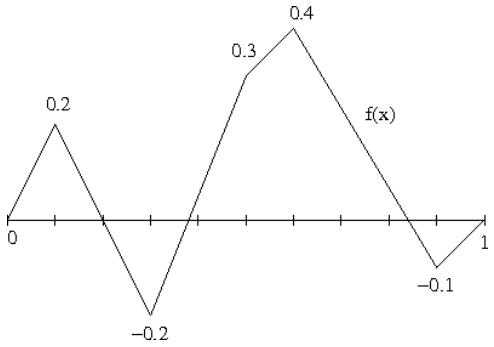
1. Solve the following problems.

   (a) Show with the help of the definition that

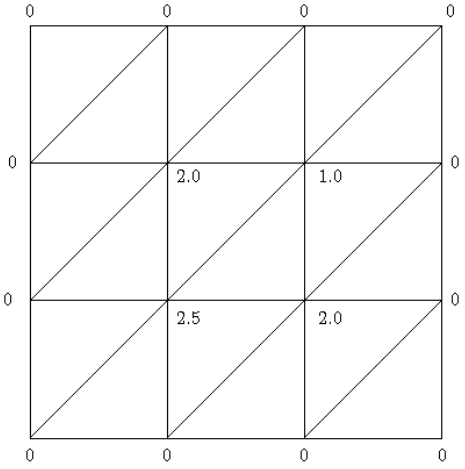
   $$f'(x) = \begin{cases} -1 & x < 0, \\ 0 & x = 0, \\ 1 & x > 0, \end{cases}$$

   is the weak derivative of $f(x) = \mid x \mid$.

   (b) Compute the weak derivative of the following function $f : \Omega \to \mathbb{R}$, $\Omega = (0, 1)$.



2. Compute the weak derivative of the following function $f : \Omega \to \mathbb{R}$, $\Omega = (0, 1)^2$



3. Let $r \in [1, \infty)$, $p, q \in (1, \infty)$, $p^{-1} + q^{-1} = 1$, $u \in L^{rp}(\Omega)$, $v \in L^{rq}(\Omega)$. Show that

$$\| u\, v \|_{L^r} \leq \| u \|_{L^{rp}} \| v \|_{L^{rq}} .$$

The exercise problems should be solved in groups of two or three students. They have to be submitted until **Tuesday, May 14, 2013** either before or after one of the lectures or directly at the office of Mrs. Hardering. The executable codes have to be send by email to Mrs. Hardering.