

**Exercises to the classes
Numerical Methods in Sciences and Technics**

Exercises no. 12

to 02.02.2004

The solution of exercise 3 is to submit in the exercise classes on Monday, 02.02.2004 !

Statements given in the lecture can be used in the solution of the exercises without proof. All other statements have to be proved.

1. Let $A \in \mathbb{R}^{n \times n}$ and let $\mu(A)$ be the logarithmic norm of A . Prove the formulae of $\mu_1(A)$ and $\mu_2(A)$ which were given in the first lemma of Section 2.1.

2. Consider the local error of the linearly implicit Euler scheme for DAEs as given in the classes. Show that

$$v_{m+1} = v_m + h (I - hf_v + hf_u g_u^{-1} g_v)^{-1} (f - f_u g_u^{-1} g),$$

where all functions on the right hand side are evaluated at $(u_m, v_m) = (u(t_m), v(t_m))$.

3. Consider the non-autonomous DAE

$$\begin{aligned} 0 &= g(t, u, v) \\ v' &= f(t, u, v). \end{aligned}$$

Show for the local errors committed with the linearly implicit Euler scheme that

$$le_u^{m+1} = \mathcal{O}(h), \quad le_v^{m+1} = \mathcal{O}(h^2).$$