

## Lösungen zu den 18. Präsenzaufgaben für MfI 2

1. Aufgabe :

(a) 4 Teilintervalle:

$$[1.0 - 1.5] : f(1.0) = 0.0 \quad f(1.5) = 1.0$$

$$[1.5 - 2.0] : f(1.5) = 1.0 \quad f(2.0) = 2.0$$

$$[2.0 - 2.5] : f(2.0) = 2.0 \quad f(2.5) = 1.0$$

$$[2.5 - 3.0] : f(2.5) = 1.0 \quad f(3.0) = 0.0$$

$$U(f) = \frac{1}{2} (0.0 + 1.0 + 1.0 + 0.0) = 1$$

$$O(f) = \frac{1}{2} (1.0 + 2.0 + 2.0 + 1.0) = 3$$

(b) 8 Teilintervalle:

$$[1.00 - 1.25] : f(1.00) = 0.0 \quad f(1.25) = 0.5$$

$$[1.25 - 1.50] : f(1.25) = 0.5 \quad f(1.50) = 1.0$$

$$[1.50 - 1.75] : f(1.50) = 1.0 \quad f(1.75) = 1.5$$

$$[1.75 - 2.00] : f(1.75) = 1.5 \quad f(2.00) = 2.0$$

$$[2.00 - 2.25] : f(2.00) = 2.0 \quad f(2.25) = 1.5$$

$$[2.25 - 2.50] : f(2.25) = 1.5 \quad f(2.50) = 1.0$$

$$[2.50 - 2.75] : f(2.50) = 1.0 \quad f(2.75) = 0.5$$

$$[2.75 - 3.00] : f(2.75) = 0.5 \quad f(3.00) = 0.0$$

$$U(f) = \frac{1}{4} (0.0 + 0.5 + 1.0 + 1.5 + 1.5 + 1.0 + 0.5 + 0.0) = \frac{3}{2}$$

$$O(f) = \frac{1}{4} (0.5 + 1.0 + 1.5 + 2.0 + 2.0 + 1.5 + 1.0 + 0.5) = \frac{5}{2}$$

2. Aufgabe :

$$\begin{aligned} A(f+g) &= \sum_{k=0}^n \binom{n}{k} (1-t)^{n-k} t^k (f+g) \left(\frac{k}{n}\right) \\ &= \sum_{k=0}^n \binom{n}{k} (1-t)^{n-k} t^k \left( f \left(\frac{k}{n}\right) + g \left(\frac{k}{n}\right) \right) \\ &= \sum_{k=0}^n \binom{n}{k} (1-t)^{n-k} t^k f \left(\frac{k}{n}\right) + (1-t)^{n-k} t^k + g \left(\frac{k}{n}\right) \\ &= A(f) + A(g) \end{aligned}$$

Analog zeigt man:  $A(\lambda f) = \lambda A(f)$