ATLAS-BASED FAST PATIENT-SPECIFIC SIMULATIONS 
OF THE PULMONARY ARTERY
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We present a novel method to enable fast patient-specific simulations, combining image analysis, computational fluid dynamics and model order reduction.

- From a preliminary set of individual geometries, a representative template (atlas) of the pulmonary artery shape is created.
- Full CFD simulations of the blood flow are carried out only on the atlas geometry. Flow information is compressed into a basis for model order reduction (proper orthogonal decomposition), creating a precomputed flow database.
- Individual patient geometries are registered to the atlas, and the precomputed basis functions are mapped onto the individual meshes, generating a patient specific reduced order flow model.

ATLAS COMPUTATION
The pulmonary artery of each patient was segmented from MR angiography. The generated meshes were then pre-processed for CFD simulations. From a preliminary mesh set, an unbiased template (atlas) of the artery shape was constructed using the forward method proposed in [1]. This approach is particularly suited for our purposes, as shapes are represented by currents, which can be used for meshes without point correspondence.

COMPUTATIONAL FLUID DYNAMICS
Using a finite element method, blood flow on the atlas geometry is simulated solving numerically the Navier-Stokes equations for blood pressure ($p$) and velocity ($\mathbf{u}$):

\[
\begin{align*}
\rho_t \partial_t \mathbf{u} + \rho (\mathbf{u} \cdot \nabla) \mathbf{u} + \nabla p - \mu \Delta \mathbf{u} &= 0 \\
\text{div} \mathbf{u} &= 0
\end{align*}
\]

Proper orthogonal decomposition
From a set of snapshots of the atlas solution at different instants of time, we compute a basis of a proper orthogonal decomposition (POD) for velocity and pressure, which consists of a reduced number of functions able to describe the fluid solution with high accuracy.

FAST PATIENT-SPECIFIC SIMULATIONS
1. Registration and 3D-deformation. The atlas surface is registered to each patient, establishing a one-to-one correspondence between surface points. Surface maps are then harmonically extended to the atlas volume mesh, describing three-dimensional patient meshes as deformation of the atlas.

\[
\begin{align*}
\Delta A &= 0, \text{ in } \Omega \\
A_{|_{\Sigma}} &= d_{\Sigma} \\
\end{align*}
\]

Fig. 2. The surface map ($d_{\Sigma}$), obtained after registration, serves as boundary condition to compute a 3D deformation $\mathbf{A}$ by harmonic extension.

2. POD mapping. Using the 3D deformation, POD basis functions can be mapped onto the individual meshes [3]. Velocity basis functions on the patient geometry ($\mathbf{w}_p$) have been computed using a Piola transform, a vector fields map preserving locally the incompressibility condition:

\[
w_{\text{patient}}(y) = \frac{1}{|\det F(x)|} F(x) w_{\text{atlas}}(x), \quad y = A(x), \quad F = \nabla A
\]

This generates patient-specific reduced models for fast individual simulations.

REFERENCES

RESULTS
The method has been validated on a set of three Tetralogy of Fallot patients. We compared full simulations, i.e. solving numerically the Navier-Stokes equations on patient geometries, and our reduced approach.

Fig. 3. Relative errors (%) between full and reduced simulations (20 POD basis functions) on three different patient geometries.

The reduced order model, with a smaller amount of unknowns, exhibits good approximation properties.

PRE-COMPUTED DATABASE
- preliminary training set
- Atlas
- CFD simulation
- POD basis
- patients geometries
- registration
- 3D deformation
- individual reduced models
- PATIENT-SPECIFIC SIMULATIONS

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