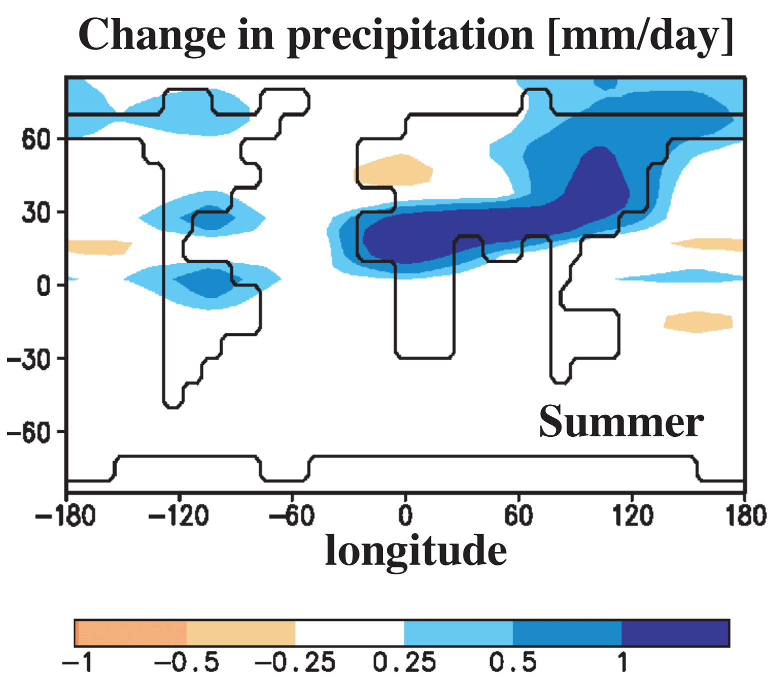
Mathematik & Informatik @ PIK

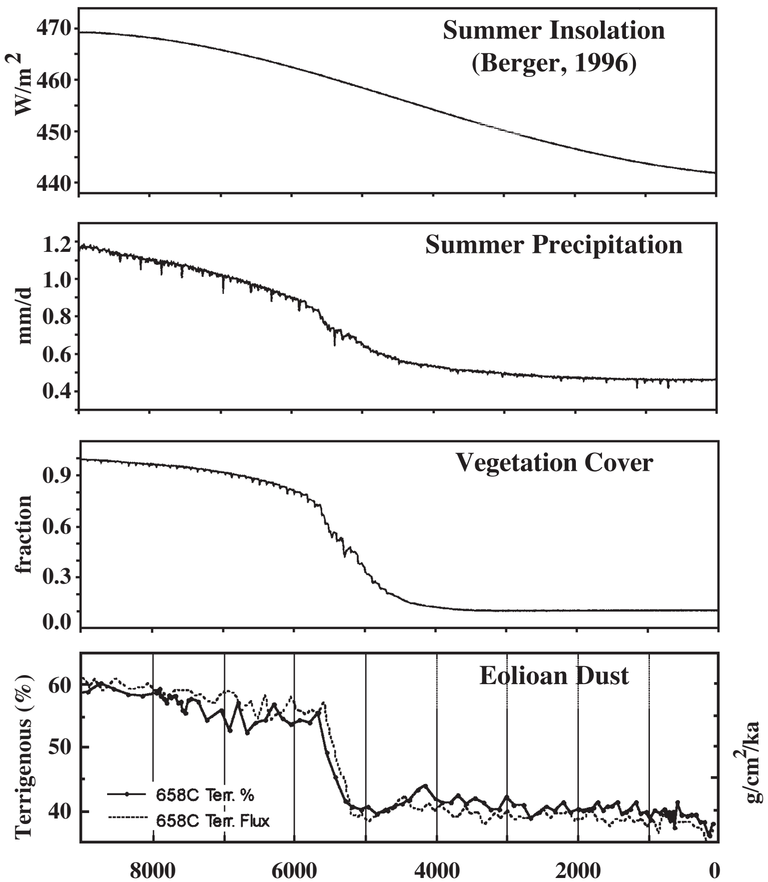
Vor ca. 5 000 Jahren:
Warum wurde die Sahara zur Wüste?



Decrease of summer precipitation
8 000 . . . 200 yr before present

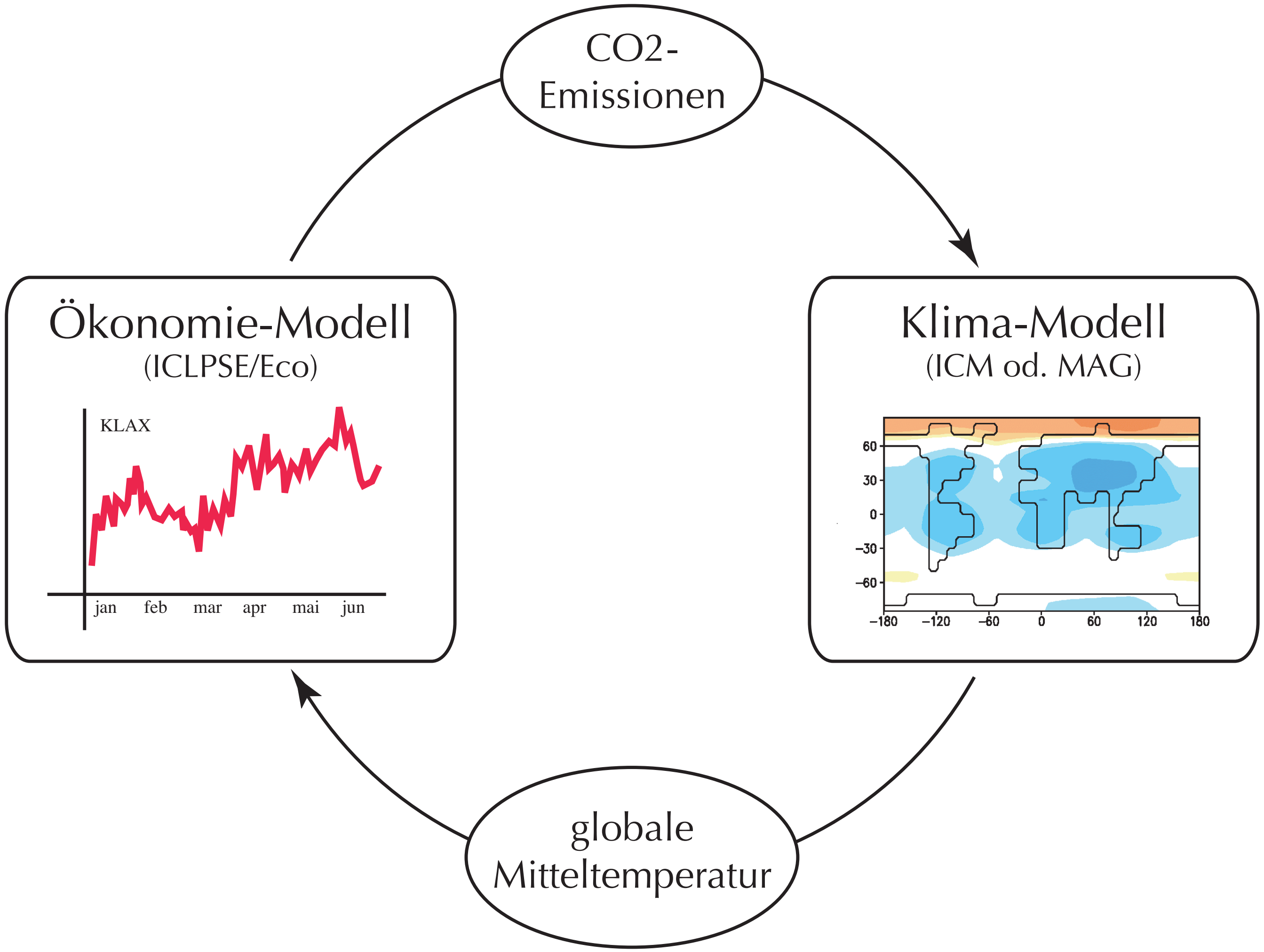
Climber-2: North Africa (20-30° N) *(Claussen et al. 1999)*

Marine core near western North Africa *(de Menocal et al. 2000)*



disziplinübergreifende Forschung

Wirtschaftliche Optimierung angesichts von Klima-Schranken



disziplinübergreifende Forschung

Mit dem globalen Erdsystem oder mit Teilen desselben

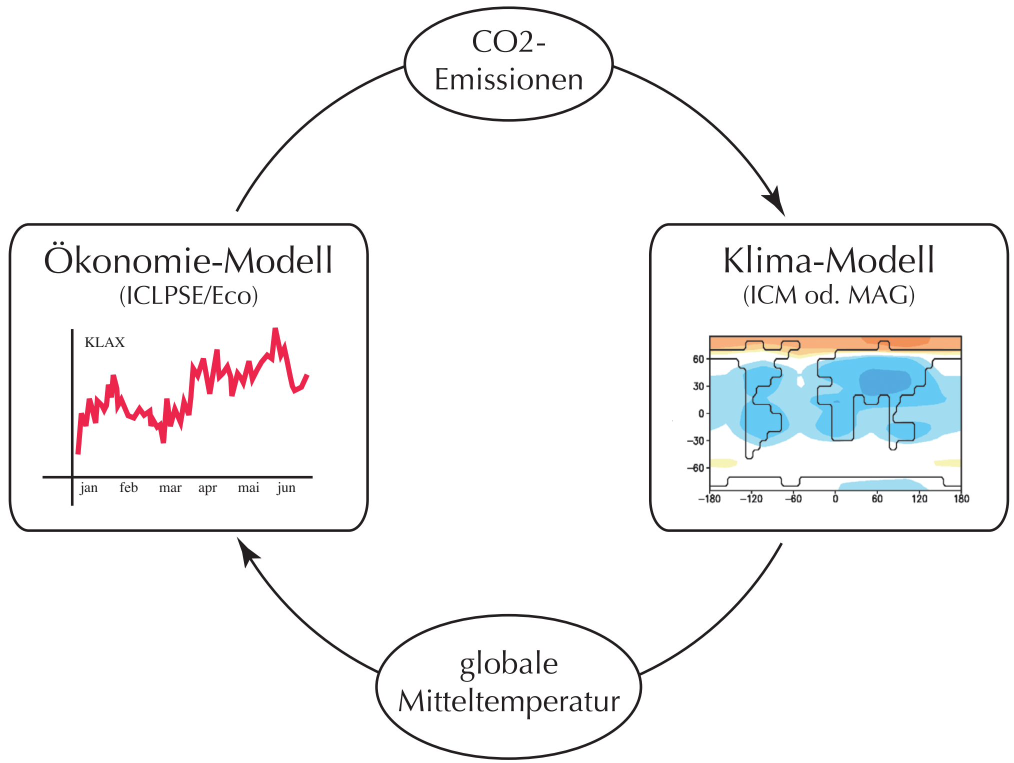
können wir nicht experimentieren.

Deshalb kommt der

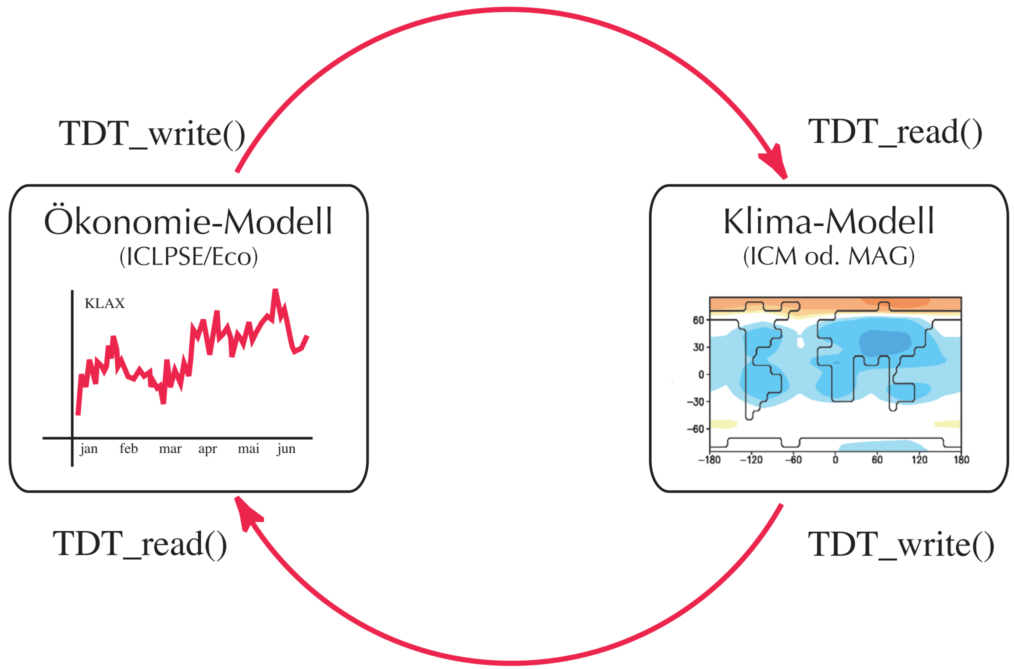
Computer-gestützten Modellierung

eine ganz zentrale Rolle zu.

Interdisziplinarität, Modularität, Modellkopplung



Interdisziplinarität, Modularität, Modellkopplung



Flexible Kopplung erfordert Datentransfer unabhängig von

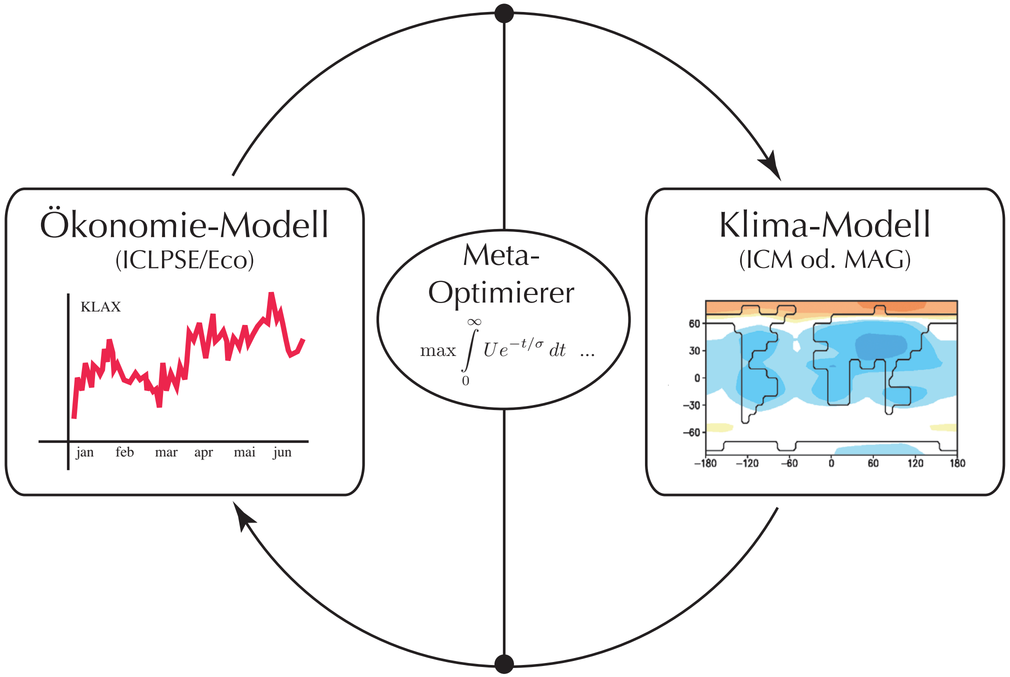
- Betriebssystem
- Hardware
- Programmiersprache

$\Longrightarrow$ **TDT**-Library

TDT (Typed Data Transfer)

Betriebssysteme	Programmiersprachen	Protokolle
FreeBSD	C/C++	Sockets
IBM AIX	FORTRAN (via C)	Files
GNU/Linux	Python	MPI
Windows 2000	**GAMS**	
Mac OS X		
gemischt		

Mathematik der gekoppelten Probleme

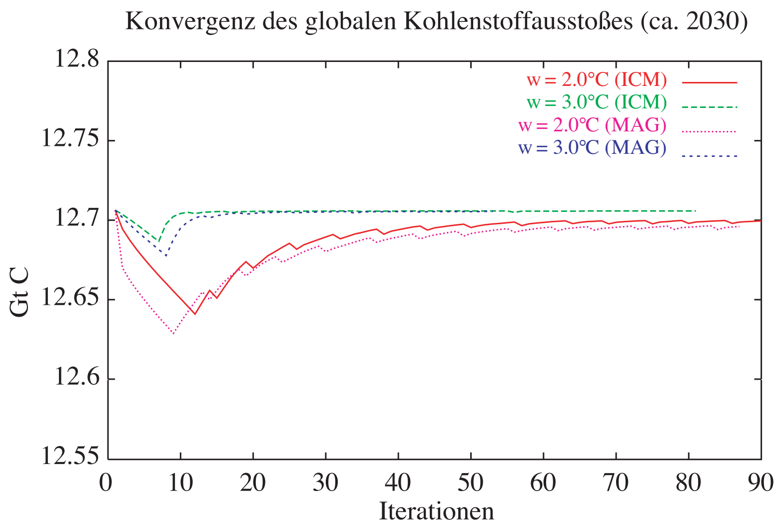


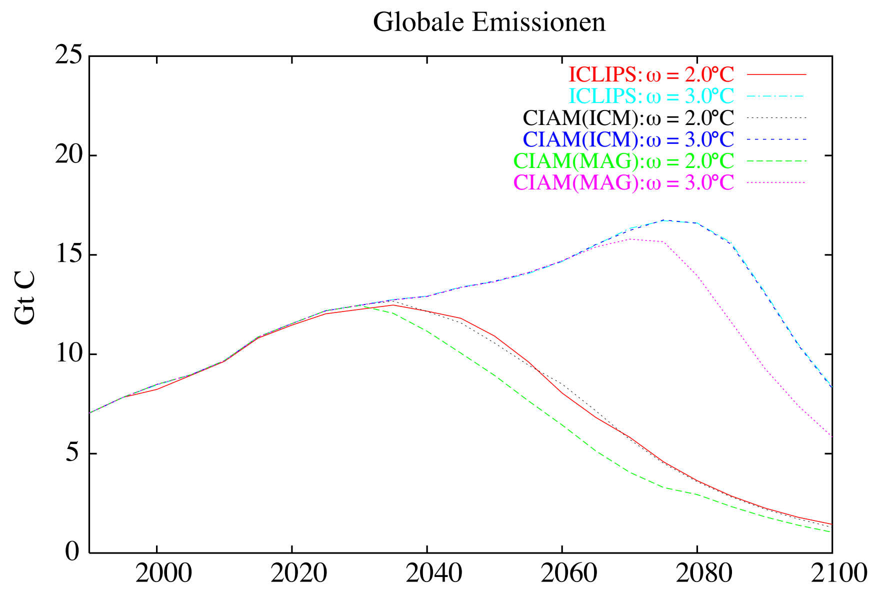
Optimierung bei minimaler struktureller Information

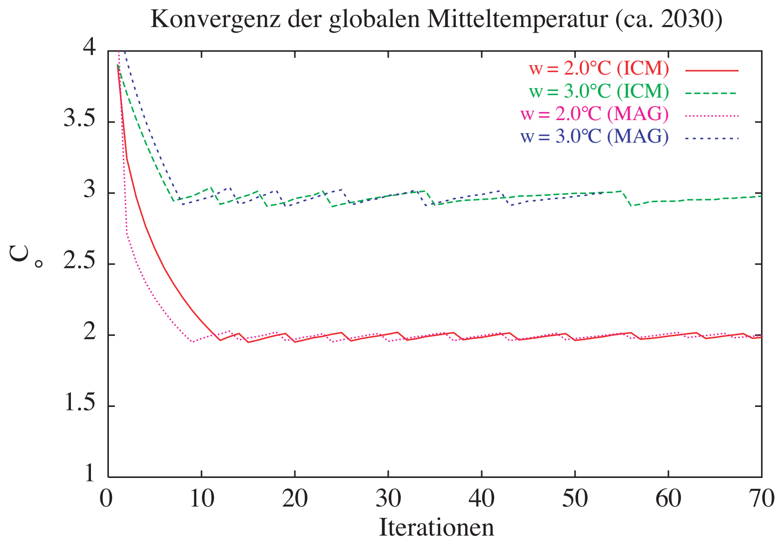
$$\max \int_0^\infty U e^{-t/\sigma}\, dt \quad \text{wo} \quad \left\{ \begin{array}{l} T < T_{\max}, \\ \dfrac{dT}{dt} < T'_{\max} \end{array} \right\} \quad \Longrightarrow$$

**Meta-
Optimierer**

Meta-Optimierung

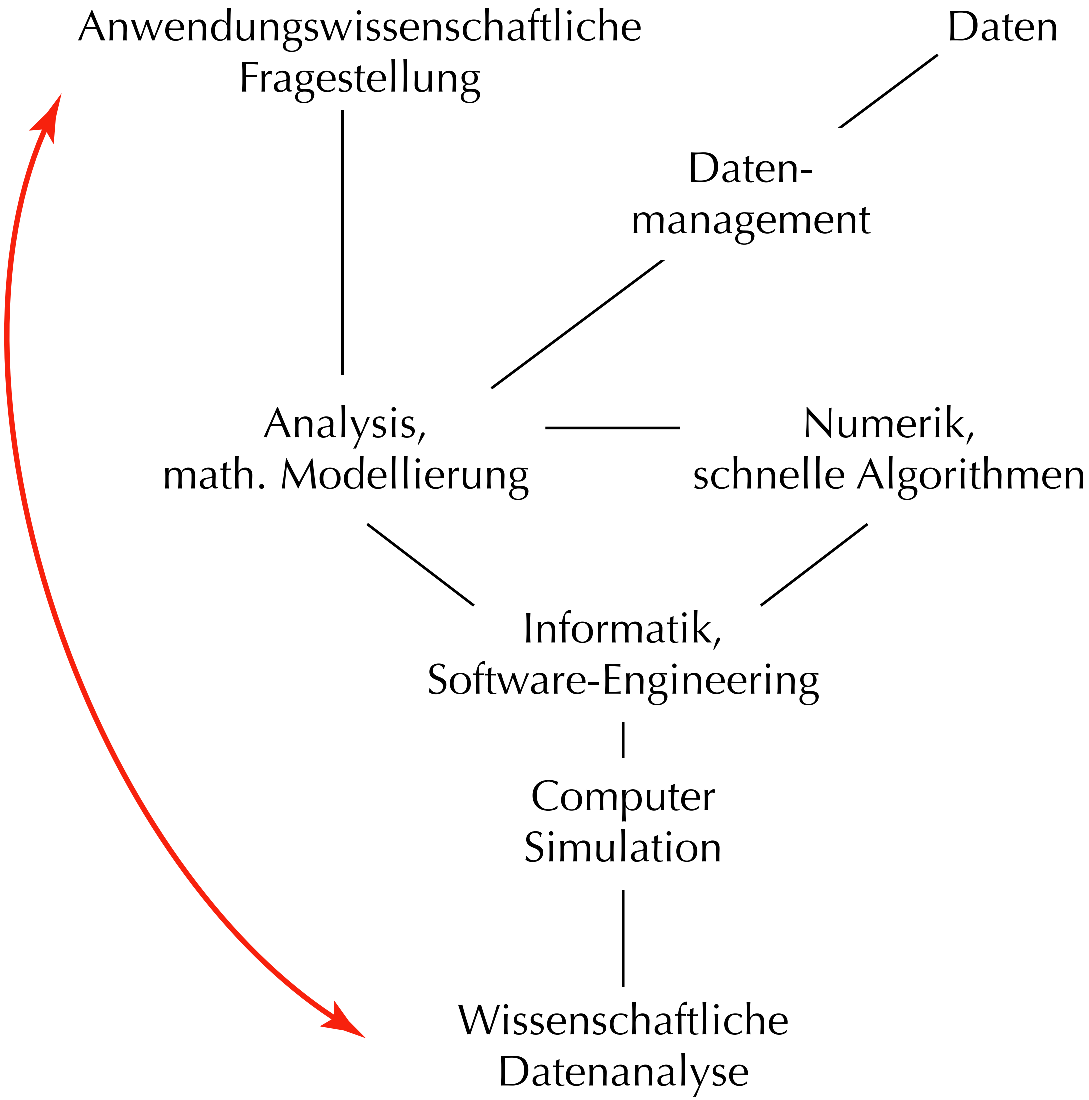






1. Startlauf mit „business as usual“.
2. CO_2-Reduktion bis Klimaschranken eingehalten sind.
3. Optimiere Ökonomie mit „weichen“ Klimaschranken.
4. zurück zu 2. bis Konvergenz erreicht ist.

Wissenschaftliches Rechnen
(Scientific Computing)



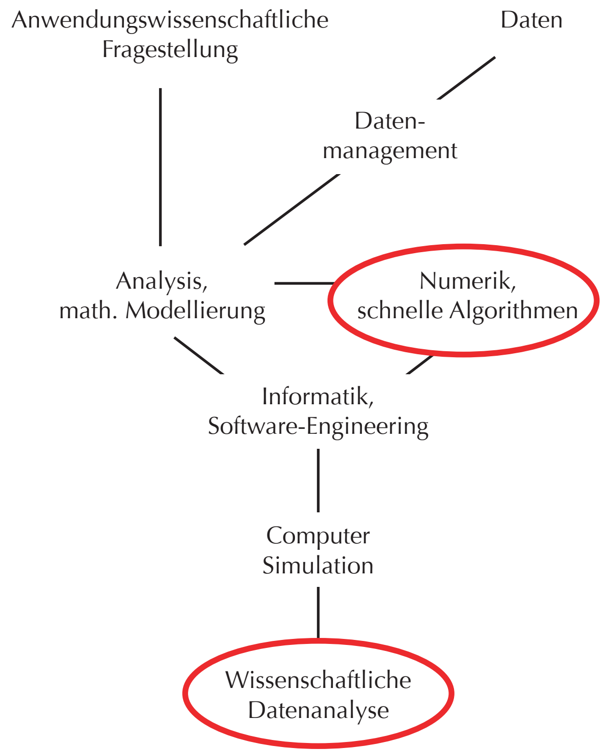
ReCSim *(mit ClimSys)*

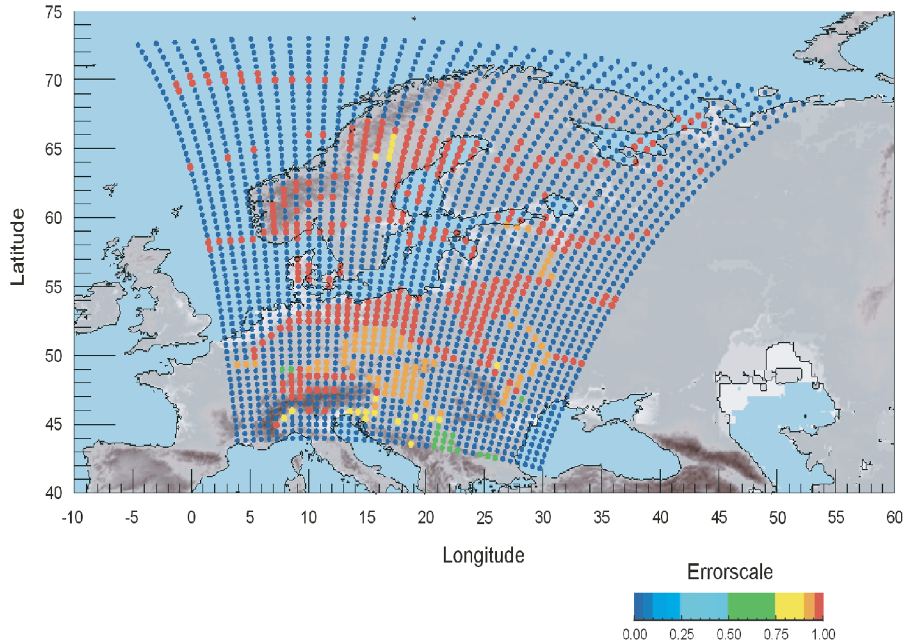
regionales ("community")
Klimamodell LM

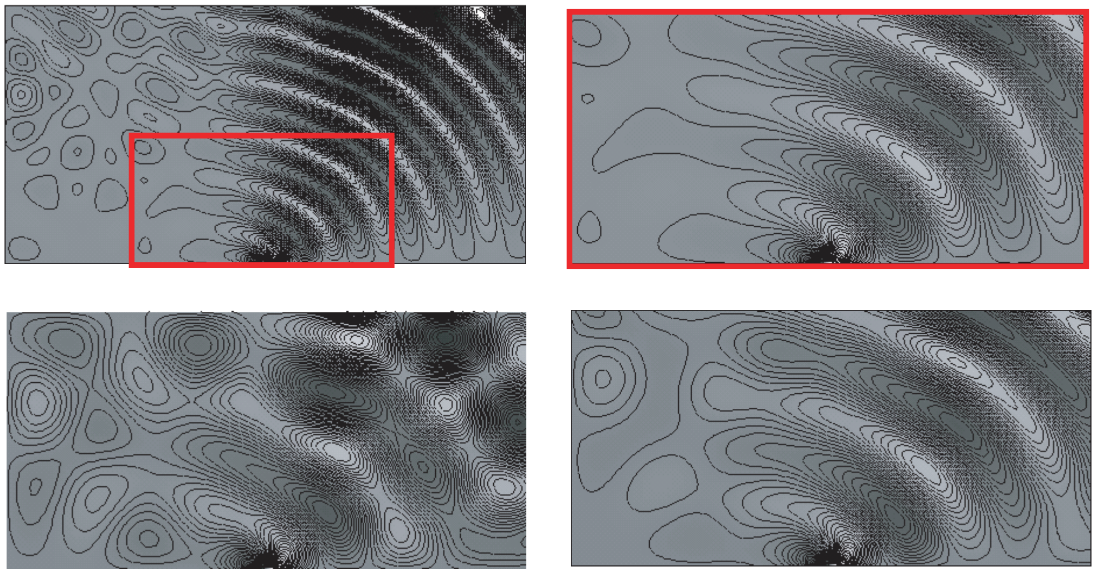
statistische Analysewerkzeuge

Modelleinbettungstechniken

Massen- und energieerhaltende Verfahren







- **PIK's Computer-System und IT-Infrastruktur**
- Computermodelle und deren Semantik
- Modellierungs- und Simulationsumgebung
- Management von Daten und Metadaten
- Strömungsberechnung in Erhaltungsform
- dynamische Modelle ökonomischer Prozesse **(mit *Abtlg. soziale Systeme*)**
- Kompakte Modelle von Wettermustern **(mit *Abtlg. Klima-System*)**
- Mathematische & technische Unterstützung bei der Modellentwicklung

(towards) Modules for Ageing Capital Stocks

Carlo Jaeger, Rupert Klein

Potsdam Institute for
Climate Impact Research

March 25, 2004

von Neumann

- Economy with a set of sectors / goods produced
- Dynamics assumes **equal growth rate for each sector**
- Result: Eigenvector of maximal (exponential) growth
- (Prices adjust accordingly)

Active vs. passive capital

- items of active capital have relatively long lifetime
- stocks of active capital have an age distribution
- stock items provide age-dependent utilities

Capital stock transition examples

- Energy plants: fossile fuel $\longrightarrow$ renewable energies
- Cities: strategic re-organization of entire quarters

Hyperbolic conservation law

$$\frac{\partial}{\partial t}\boldsymbol{K} + \frac{\partial}{\partial \tau}\boldsymbol{F}(\boldsymbol{K}) = 0$$

$$\boldsymbol{F}(\boldsymbol{K}) = \boldsymbol{K}$$

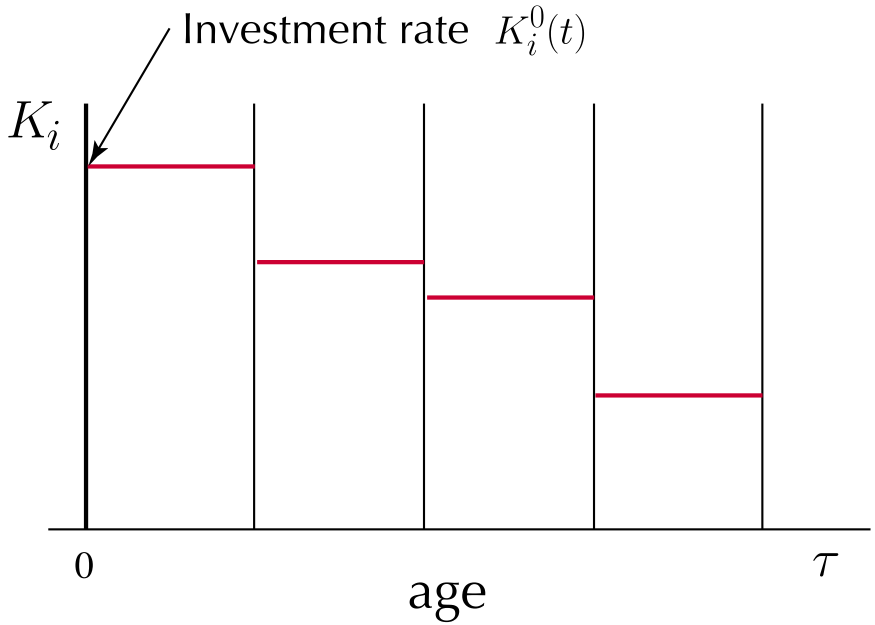
Initial / Boundary conditions

$$\boldsymbol{K}(\tau, 0) = \boldsymbol{K}_0(\tau)$$

$$\boldsymbol{K}(0, t) = \boldsymbol{K}^0(t)$$

Characteristic form

$$\frac{\partial}{\partial t}\boldsymbol{K} + \frac{\partial}{\partial \tau}\boldsymbol{K} = 0$$



Capital age densities

$$\boldsymbol{K} = (K_0, \ldots, K_{n-1})$$

Integral conservation law

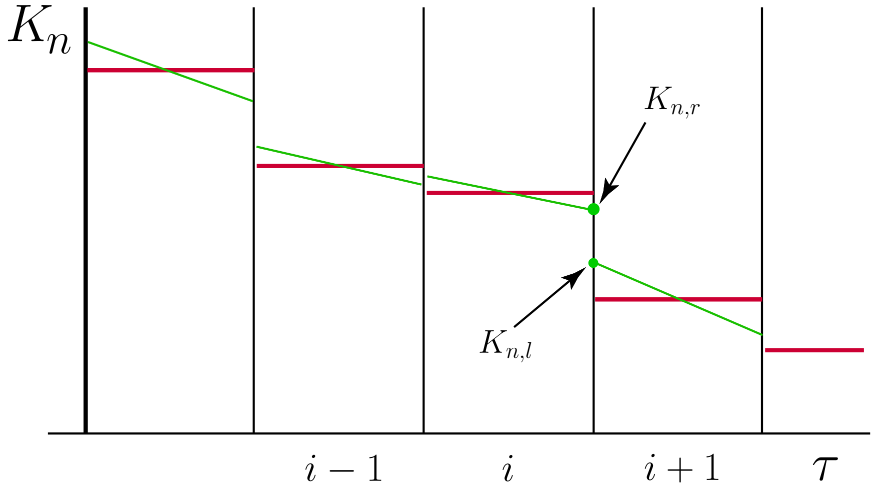
$$\int_{\tau_{i-1/2}}^{\tau_{i+1/2}} \boldsymbol{K}(\tau,t)\,d\tau = -\left(\int_{t_n}^{t_{n+1}} \boldsymbol{F}(\boldsymbol{K}(\tau_{i+1/2},t))\,dt - \int_{t_n}^{t_{n+1}} \boldsymbol{F}(\boldsymbol{K}(\tau_{i-1/2},t))\,dt \right)$$

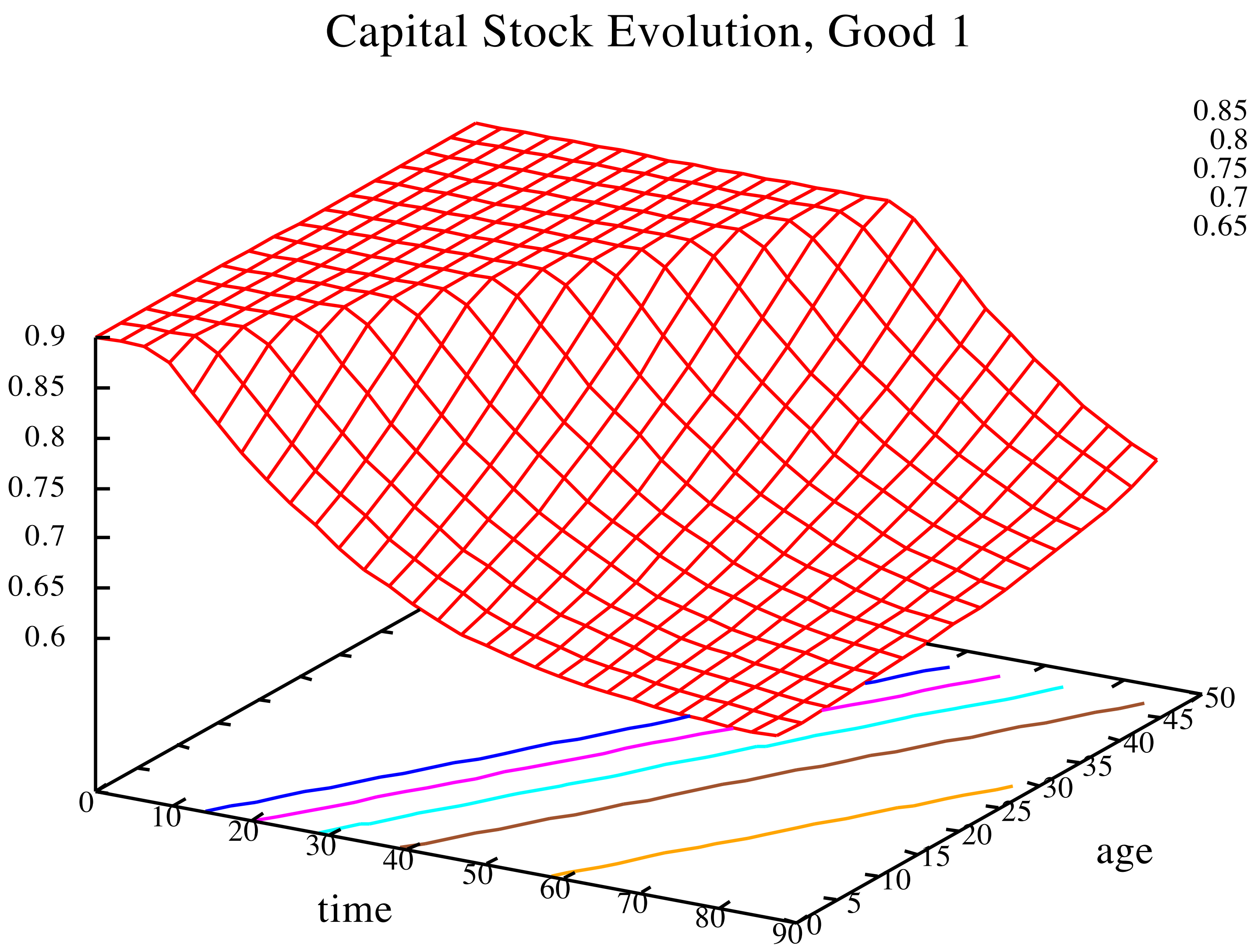
1. space-time reconstruction
2. numerical flux functions

$$\boldsymbol{F}_{i+1/2} \approx \boldsymbol{F}^{\text{num.}}(\boldsymbol{K}_l, \boldsymbol{K}_r)$$

3. approximate time integration, e.g.,

$$\boldsymbol{K}_i^{\nu+1} - \boldsymbol{K}_i^{\nu} = -\frac{\Delta\tau}{\Delta t}\left(\boldsymbol{F}_{i+1/2} - \boldsymbol{F}_{i-1/2}\right)^{\nu+1/2}$$




Capital Stock Evolution, Good 1
0.85
0.8
0.75
0.7
0.65
0.9
0.85
0.8
0.75
0.7
0.65
0.6
0
10
20
30
40
50
60
70
80
90
time
0
5
10
15
20
25
30
35
40
45
50
age

- **Transition management under constraints from**
  - invest budget
  - desired final distribution of goods
  - same with economically determined budgets
  - same with time dependent penalties for "obsolete goods"

- **Interface to capital market**
  - "active" vs. "passive" capital
  - non-zero time scales of "active" capital included
  - budget constraints: from investment bank model

- **Interdependence of goods**

- ...

Possible Embeddings in an IAM

$$\boldsymbol{K} = (K_0, K_1)$$

$$\max_{\boldsymbol{K}^0} \left\{ \int_{t=0}^{t_{\max}} \int_{\tau=0}^{\infty} \boldsymbol{u}\left(\boldsymbol{K}(t,\tau), t, \tau\right) \quad d\tau\, dt \right\}$$

Capital stock evolution

$$\frac{\partial}{\partial t}\boldsymbol{K} + \frac{\partial}{\partial \tau}\boldsymbol{K} = 0 \qquad \left\{ \begin{array}{rcl} \boldsymbol{K}(\tau,0) & = & \boldsymbol{K}_0(\tau) \\ \boldsymbol{K}(0,t) & = & \boldsymbol{K}^0(t) \end{array} \right.$$

Constraints

$$K_i^0(t) \geq 0 \qquad \text{positivity}$$

$$\sum_{i=0}^{1} P_i K_i^0(t) \leq \boldsymbol{B}(t) \qquad \text{invest budget}$$

$$\frac{\int_0^{\tau_{\max}} K_0(\tau,T)\ d\tau}{\int_0^{\tau_{\max}} K_1(\tau,T)\ d\tau} \leq \boldsymbol{Q}(\tau) \qquad \text{targeted final distribution}$$

Utility

$$\boldsymbol{u}\left(\boldsymbol{K}(t,\tau),t,\tau\right)=\sum_{\boldsymbol{i=0}}^{\mathbf{1}}\int_0^T \boldsymbol{u}_{\mathrm{time}}(t)\;\boldsymbol{u}_{\boldsymbol{i},\mathrm{mass}}\left(\int_0^{\tau_{\max}}\boldsymbol{u}_{\boldsymbol{i},\mathrm{age}}(\tau)K_{\boldsymbol{i}}(\tau,t)\,d\tau\right)\,dt$$

Jelly Stock Evolution

Solves capital stock initial / boundary value problem

addtive Utilities

utility as fct. of age and amount

discounting in time

Constraints

positivity

budget constraints

target distributions

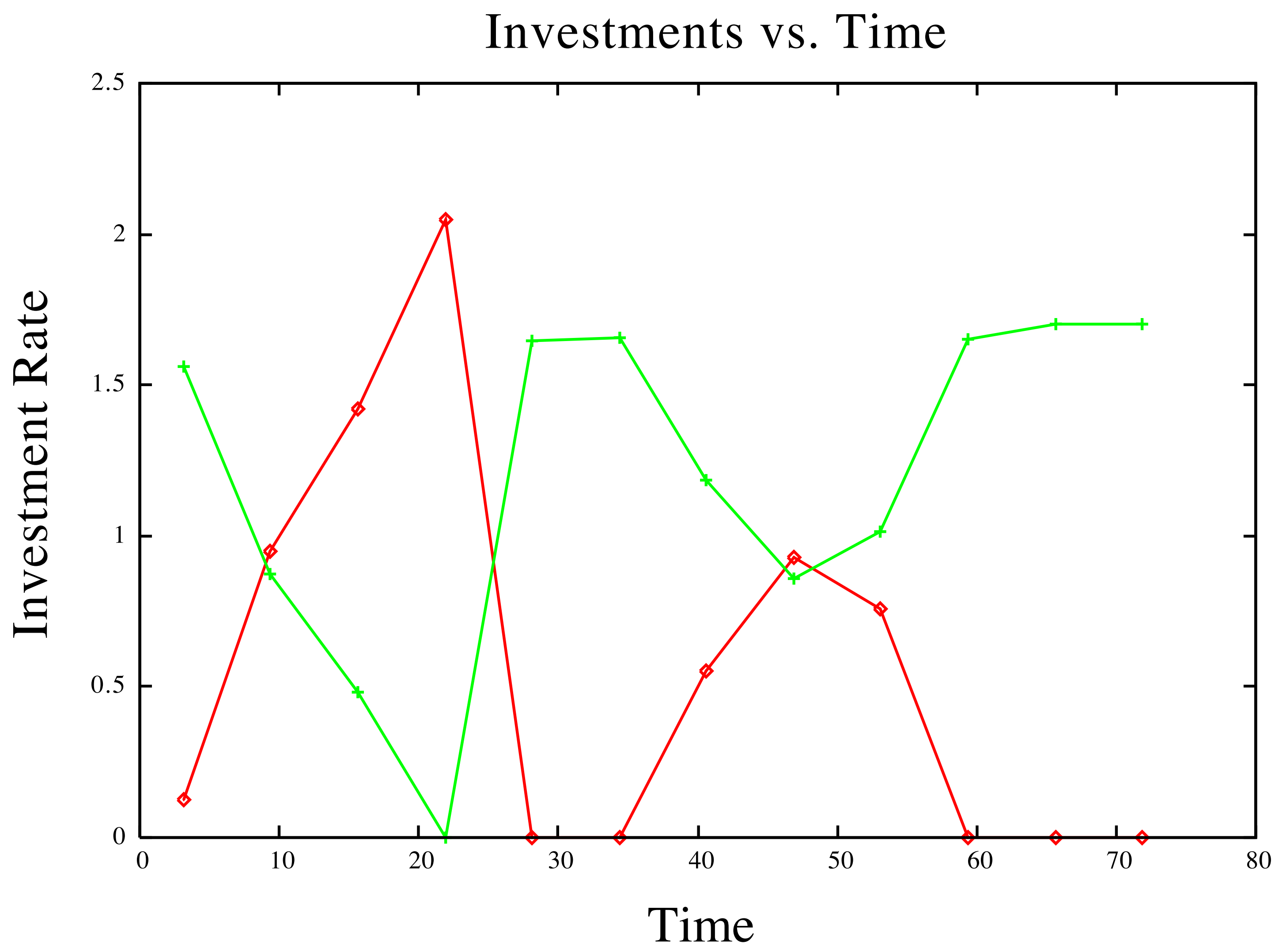
Numerical Coupling

“smart hare” optimization

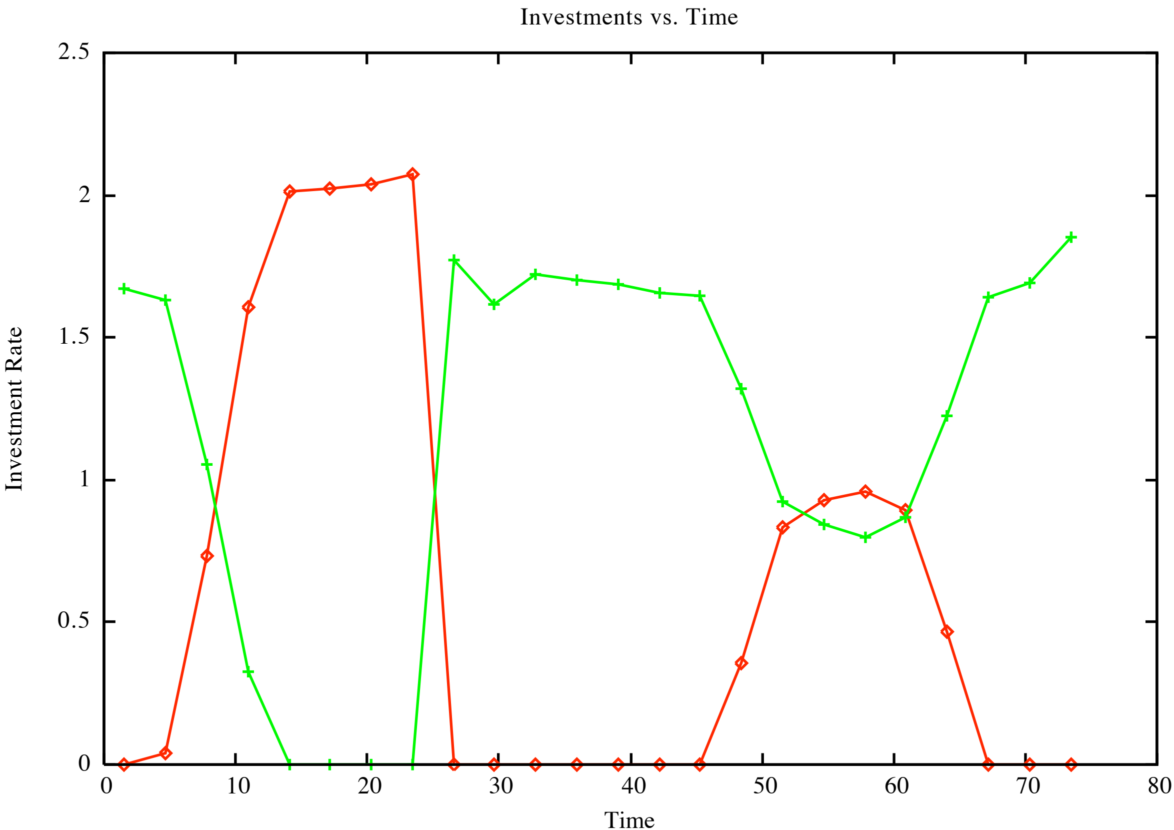
random search

. . .

Example: Modules



Example: Preliminary Result



Investments vs. Time
Investment Rate
Time
0
0.5
1
1.5
2
2.5
0
10
20
30
40
50
60
70
80